Supplementary Document:

Innovation and Technology Diffusion in Competitive Supply Chains

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Appendices

A. Asymmetric Downstream Markets

When the downstream firms are asymmetric in terms of idiosyncratic valuations, cross- and self-price or capability sensitivities, and adoption cost functions, a stylized treatment of the game discussed in this paper is not practical. In this section, we discuss the numerical analysis of 2,3,4-firm downstream tiers facing linear demand functions - (L) - asymmetric in a subset of these factors. The details of the algorithm (LDCEA) used to solve the equilibrium of the whole game is provided in Appendix A.1.

We show that the majority of the properties of the upstream return function and downstream technological potential previously seen in the symmetrical system are preserved.

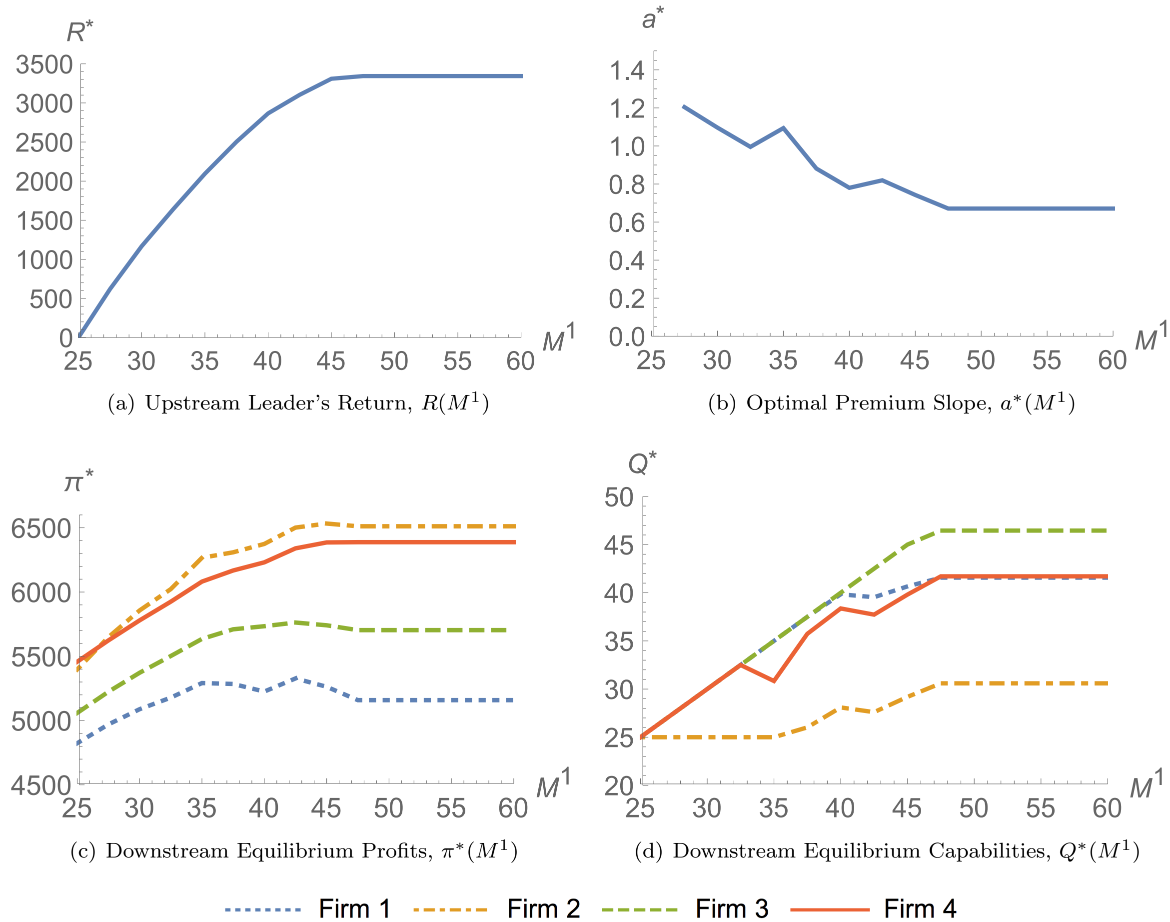
We construct the exact upstream leader's return curve, $R(M^1)$, for all $M^1 \geq M^2$, in addition to the optimal premium slope, $a^*(M^1)$, equilibrium downstream capability levels Q_j^*, and downstream profits π_j^* as a function of M^1. The results of these asymmetric cases are then compared to the symmetric base cases from which they are derived. Due to the immense number of possibilities of asymmetries, we do not give a complete account for how different asymmetry types affect different equilibrium variables. We point out some observations which are robust even under multiple types of asymmetry. A representative example is provided below.

Example:

$n = 4$, $M^2 = 25$, $\alpha = (30\ 40\ 40\ 50)^\top$, $\kappa = (3\ 2\ 2.5\ 2)^\top$, $Q_j^0 = 5\ \forall j$, $c = (0.1\ 0.1\ 0.1\ 0.15)$.

$$\beta = \begin{pmatrix} -0.6 & 0.3 & 0.3 & 0.3 \\ 0.3 & -0.8 & 0.3 & 0.3 \\ 0.3 & 0.2 & -0.6 & 0.2 \\ 0.2 & 0.2 & 0.4 & -0.8 \end{pmatrix}, \gamma = \begin{pmatrix} 2.2 & -0.6 & -0.6 & -0.6 \\ -0.5 & 1.9 & -0.4 & -0.4 \\ -0.5 & -0.4 & 2 & -0.4 \\ -0.5 & -0.4 & -0.6 & 2.1 \end{pmatrix}.$$

The TP of this 4-firm downstream tier is $\tau \sim (42.94\ \ 43.38\ \ 59.13\ \ 65.46)^\top$.



(a) Upstream Leader's Return, $R(M^1)$

(b) Optimal Premium Slope, $a^*(M^1)$

(c) Downstream Equilibrium Profits, $\pi^*(M^1)$

(d) Downstream Equilibrium Capabilities, $Q^*(M^1)$

Figure .1: An Asymmetric Downstream Tier with Four Firms Facing Linear Demand: Equilibrium as a Function of Upstream Leader's Capability

Observation 1. *The upstream leader's return is a quasi-concave increasing function in M^1 between the upstream laggard capability (M^2) and maximum pass-through level ($\bar{Q}(M^2)$), and constant for capability levels above $\bar{Q}(M^2)$.*

Observation 2. *If the downstream firms are asymmetric in some parameter, the optimal premium slope a^* is not necessarily a monotonic function of upstream capability (Figure 1(b)). The upstream leader may increase or reduce a^* with its increasing capability. This results in non-monotonic levels of adoption by downstream firms (Figure 1(d)). Downstream firms' equilibrium profits are also possibly not monotonic in the upstream leader's capability (Figure 1(c)).*

Observation 3. *If the downstream firms are symmetric in all senses, the optimal premium slope is a monotonic decreasing function of upstream leader's capability and the downstream firms' adoption levels and profits are monotonically increasing in the upstream leader's capability.*

M^1	$a^*(M^1)$	$\mathbf{Q}^{*\top}$	$\pi^{*\top}$	$R(M^1)$
25	0.000	(25.000 25.000 25.000 25.000)	(4823.74 5397.54 5059.28 5457.52)	0.00
27.5	1.204	(27.500 25.000 27.500 27.500)	(4969.98 5649.03 5224.13 5623.51)	619.83
30	1.096	(30.000 25.000 30.000 30.000)	(5088.23 5856.61 5371.72 5779.27)	1167.55
32.5	0.995	(32.500 25.000 32.500 32.499)	(5178.29 6022.56 5502.01 5924.80)	1641.90
35	1.094	(35.000 25.000 35.000 30.834)	(5291.76 6266.41 5635.09 6081.09)	2094.53
37.5	0.881	(37.500 26.020 37.500 35.735)	(5284.80 6308.94 5709.92 6167.50)	2503.57
40	0.780	(39.840 28.091 40.000 38.369)	(5225.65 6374.14 5734.04 6230.87)	2866.46
42.5	0.819	(39.537 27.602 42.500 37.727)	(5327.66 6501.34 5762.63 6339.77)	3104.40
45	0.742	(40.658 29.188 45.000 39.807)	(5262.75 6532.79 5741.15 6385.86)	3309.21
47.5	0.671	(41.567 30.593 46.454 41.705)	(5158.77 6511.57 5703.55 6387.84)	3343.44
50	0.671	(41.567 30.593 46.454 41.705)	(5158.77 6511.57 5703.55 6387.84)	3343.44
52.5	0.671	(41.567 30.593 46.454 41.705)	(5158.77 6511.57 5703.55 6387.84)	3343.44
55	0.671	(41.567 30.593 46.454 41.705)	(5158.77 6511.57 5703.55 6387.84)	3343.44
57.5	0.671	(41.567 30.593 46.454 41.705)	(5158.77 6511.57 5703.55 6387.84)	3343.44
60	0.671	(41.567 30.593 46.454 41.705)	(5158.77 6511.57 5703.55 6387.84)	3343.44

Table .1: Optimal premium slope, equilibrium capabilities and profits.

For most practical purposes, the return to a superior upstream technology from the downstream tier implied by the capability dimension, $R(M^1)$, can be approximated with a concave or quasi-concave increasing function up to a capability level (which is lower than the highest element of the technological potential) and constant for any capability level beyond. Any technology possessed above this level, $\bar{Q}(M^2)$, is "held-up" in the upper tier. This approximation is very similar to the function depicted in Figure 2(a) for the symmetrical case. Thus, the symmetrical analysis constructed in §4.3 is a good proxy for the asymmetric case.

A.1. LDCEA Algorithm (For §A)

1. Initialization: Define an indicator state vector $\mathbf{s}$ for the downstream capability equilibrium $\ni s_j = \begin{cases} 1 & \text{if } Q_j < Q_j^0, \\ 2 & \text{if } Q_j = Q_j^0, \\ 3 & \text{if } Q_j \in (Q_j^0, M^{[2]}), \\ 4 & \text{if } Q_j = M^{[2]}, \\ 5 & \text{if } Q_j \in (M^{[2]}, M^{[1]}), \\ 6 & \text{if } Q_j = M^{[1]}. \end{cases}$

   Initialize the set $\mathcal{S}$ of all possible states (6^n elements). Also define:

$$J = \begin{pmatrix} 1 & ... & 1 \\ 1 & ... & 1 \\ 1 & ... & 1 \end{pmatrix}_{n\times n}, \mathbb{1}(\mathbf{s} \geq i) = \begin{pmatrix} \mathbb{1}(s_1 \geq i) \\ ... \\ \mathbb{1}(s_n \geq i) \end{pmatrix}, \kappa = \begin{pmatrix} \kappa_1 & 0... & \\ 0... & \kappa_j & 0... \\ 0... & 0... & \kappa_n \end{pmatrix},$$

   $\mathbf{c} = (c_1 \, ...c_j \, ...c_n)^\top$,

$$diag(\mathbb{1}(\mathbf{s} \geq 5)) = \begin{pmatrix} \mathbb{1}(s_1 \geq 5) & ... & 0 \\ 0 & \mathbb{1}(s_j \geq 5) & 0 \\ 0 & ... & \mathbb{1}(s_n \geq 5) \end{pmatrix}.$$

To be used for later in check for the possibility of multiple equilibria under any state $\mathbf{s}$, also create the vectors $dm(\mathbf{s})$, $dD(\mathbf{s})$; and the matrices $\triangledown m(\mathbf{s})$, and $\triangledown D(\mathbf{s})$:

$$\begin{aligned} dm(\mathbf{s})_j &= \tilde{\gamma}_{jj} + a(\tilde{d}_{jj} - 1)\mathbb{1}(s_j \geq 5), \\ dD(\mathbf{s})_j &= \hat{\gamma}_{jj} + a\ \hat{d}_{jj}\mathbb{1}(s_j \geq 5), \\ \triangledown m(\mathbf{s})_{jj} &= 0, \\ \triangledown D(\mathbf{s})_{jj} &= 0, \end{aligned}$$

$$\begin{aligned} \triangledown m(\mathbf{s})_{jk} &= \tilde{\gamma}_{jk} + a\tilde{d}_{jk}\mathbb{1}(s_k \geq 5), \\ \triangledown D(\mathbf{s})_{jk} &= \hat{\gamma}_{jk} + a\hat{d}_{jk}\mathbb{1}(s_k \geq 5). \end{aligned}$$

2. For all $\mathbf{s} \in \mathcal{S}$:

   i. Check if multiple equilibria is expected. If

$$dm(\mathbf{s}) * (\triangledown D(\mathbf{s}).\mathbf{e}) + dD(\mathbf{s}) * (\triangledown m(\mathbf{s}).\mathbf{e}) - dm(\mathbf{s}) * dD(\mathbf{s}) + 2\kappa > 0$$

   holds, there is a unique equilibrium. Otherwise, there may be multiple.

   ii. Create the candidate equilibrium with $\mathbf{Q}^{cand}(\mathbf{s}) = A^{-1}(\mathbf{s}) \cdot b(\mathbf{s})$, where:

$$A_{j.}(\mathbf{s}) = \begin{cases} \Upsilon_{j.}(\mathbf{s}) & \text{if } s_j \in \{1,3,5\}, \\ \mathbf{e}_j & \text{if } s_j \in \{2,4,6\}. \end{cases} \tag{.1}$$

$$b_j(\mathbf{s}) = \begin{cases} Z_j(\mathbf{s}) & \text{if } s_j \in \{1,3,5\}, \\ Q_j^0 & \text{if } s_j = 2, \\ M^{[2]} & \text{if } s_j = 4, \\ M^{[1]} & \text{if } s_j = 6, \end{cases} \tag{.2}$$

where

$$\begin{aligned}\Upsilon(\mathbf{s}) = dp(\mathbf{s}) * \hat{\gamma} + dD(\mathbf{s}) * \tilde{\gamma} - 2.\mathbb{1}(\mathbf{s} \geq 3) * \kappa \\ + adp(\mathbf{s}) * \hat{d} * (diag(\mathbb{1}(\mathbf{s} \geq 5).J)^\top \\ + adD(\mathbf{s}) * \tilde{d} * (diag(\mathbb{1}(\mathbf{s} \geq 5).J)^\top \\ - a.diag(dD(\mathbf{s}) * \mathbb{1}(\mathbf{s} \geq 5)),\end{aligned} \tag{.3}$$

$$\begin{aligned}Z(\mathbf{s}) = dD(s) * \mathbf{c} + c_u.dD(s) - dp(\mathbf{s}) * \hat{\alpha} - dp(\mathbf{s}) * \hat{\epsilon} - dD(\mathbf{s}) * \tilde{\alpha} - dD(\mathbf{s}) * \tilde{\epsilon} \\ + aM^2 dp(\mathbf{s}) * \hat{d}.\mathbb{1}(\mathbf{s} \geq 5) \\ + aM^2 dD(\mathbf{s}) * \tilde{d}.\mathbb{1}(\mathbf{s} \geq 5) \\ - aM^2 dD(\mathbf{s}) * \mathbb{1}(\mathbf{s} \geq 5)\end{aligned} \tag{.4}$$

where

$$\begin{aligned}dp(\mathbf{s}) &= Diag(\tilde{\gamma}) + a\left(Diag(\tilde{d}) - \mathbf{e} * \mathbb{1}(\mathbf{s} \geq 5)\right), \\ dD(\mathbf{s}) &= Diag(\hat{\gamma}) + aDiag(\hat{d}) * \mathbb{1}(\mathbf{s} \geq 5)\end{aligned}$$

iii. Check consistency of FOCs for $\mathbf{Q}^{cand}(\mathbf{s})$.

For all j:

i. if $s_j = 2$ check if $\zeta_j^1(s_{-j}, \mathbf{Q}^{cand}) \geq 0$ and $\zeta_j^3(s_{-j}, \mathbf{Q}^{cand}) \leq 0$,
ii. if $s_j = 4$ check if $\zeta_j^3(s_{-j}, \mathbf{Q}^{cand}) \geq 0$ and $\zeta_j^5(s_{-j}, \mathbf{Q}^{cand}) \leq 0$,
iii. if $s_j = 6$ check if $\zeta_j^5(s_{-j}, \mathbf{Q}^{cand}) \geq 0$,
iv. if $s_j = 1$, check if $Q_j^{cand} < Q_j^0$,
v. if $s_j = 3$, check if $Q_j^{cand} \in (Q_j^0, M^{[2]})$,
vi. if $s_j = 5$, check if $Q_j^{cand} \in (M^{[2]}, M^{[1]})$.

where

$$\zeta_j^k(s_{-j}, \mathbf{Q}) = \Upsilon(s_j = k, s_{-j}).\mathbf{Q} - Z(s_j = k, s_{-j}), \quad \forall k \in \{1, 3, 5\}. \tag{.5}$$

If sustained for all the elements of $\vec{s}$, then record $\mathbf{Q}^{cand}(\mathbf{s})$. Otherwise discard.

B. Proofs of Some Propositions and Corollaries

Proof of Proposition 1

(G) We need to show that $\mathbf{p}$ that simultaneously solves the FOCs in (3) exists, is unique, and returns non-negative profit to each downstream firm. Let $p_j^{\mu}(p_{-j})$ denote the marginal best responses which satisfy the FOC in (3) of the concave problem:

$$\max_{p_j} \, (p_j - c_j - c_u - \delta(Q_j))D_j(\mathbf{Q}, \mathbf{p}). \tag{.6}$$

Since $\partial D_j(\mathbf{Q}, \mathbf{p})/\partial p_j \leq 0$, $p_j^{\mu}(p_{-j}) \geq c_j + c_u + \delta(Q_j)$. Hence, the price best response which also returns non-negative profit, denote it with $p_j(p_{-j})$, is equal to the marginal price response, i.e., $p_j(p_{-j}) = p_j^{\mu}(p_{-j})$. By implicit derivation the slope of marginal price response of a firm j to that of firm k, $k \neq j$:

$$\frac{\partial p_j(p_{-j})}{\partial p_j} = -\frac{\dfrac{\partial D_j(\mathbf{Q},\mathbf{p})}{\partial p_k} + (p_j - c_j - c_u - \delta(Q_j))\dfrac{\partial^2 D_j(\mathbf{Q},\mathbf{p})}{\partial p_j \partial p_k}}{2\dfrac{\partial D_j(\mathbf{Q},\mathbf{p})}{\partial p_j} + (p_j - c_j - c_i - \delta(Q_j))\dfrac{\partial^2 D_j(\mathbf{Q},\mathbf{p})}{(\partial p_j)^2}} \tag{.7}$$

Under Assumption 1 (a) and (b): $\partial p_j(p_{-j})/\partial p_k \geq 0, \forall j, k \neq j$. Hence, $\mathbf{p}^*(\mathbf{Q}|\,\delta(.))$ exists and is unique.

(L) For the demand form in Equation (1) FOCs in (3) can be reorganized into:

$$\alpha_j + \beta_{j.} \cdot \mathbf{p} + \beta_{jj}\vec{p} + \gamma_{j.} \cdot \mathbf{Q} - \beta_{jj}(c_j + c_u) - \beta_{jj}\delta(Q_j) = 0, \; \forall j. \tag{.8}$$

Define $B = \beta + \mathbf{Diag}(\beta) \cdot I$, $C = \mathbf{Diag}(\beta) * (\vec{c} + c_u.I)$, and $\nabla(\mathbf{Q}) = (\beta_{11}\delta(Q_1) \, ... \, \beta_{jj}\delta(Q_j) \, ... \, \beta_{nn}\delta(Q_n))^{\top}$. Then, (.8) can be rewritten in the following matrix form:

$$\alpha + B\mathbf{p} + \gamma\mathbf{Q} - C - \nabla(\mathbf{Q}) = 0. \tag{.9}$$

Solving for $\mathbf{p}$ (B is invertible under Assumption 1):

$$\mathbf{p}^*(\mathbf{Q}|\,\delta(.)) = \underbrace{-B^{-1}\alpha}_{\tilde{\alpha}} \underbrace{-B^{-1}\gamma}_{\tilde{\gamma}} \mathbf{Q} + \underbrace{B^{-1}C}_{\tilde{\epsilon}} + B^{-1}\nabla(\mathbf{Q}), \tag{.10}$$

where $B^{-1}\nabla(\mathbf{Q}) = B^{-1} * [\mathbf{Diag}(\beta) * e]^{-1} = \tilde{d}\delta(\mathbf{Q})$.

(M) For the demand form in Equation (2) FOCs in (3) can be rewritten as:

$$D_j(\mathbf{Q}, \mathbf{p}) + (p_j - c_j - c_u - \delta(Q_j)) \underbrace{\frac{\partial D_j(\mathbf{Q}, \mathbf{p})}{\partial Q_j}}_{\beta_{jj} D_j(\mathbf{Q}, \vec{p})} = 0,$$

$$D_j(\mathbf{Q}, \mathbf{p})(1 - \beta_{jj}(p_j - c_j - c_u - \delta(Q_j))) = 0,$$

Solving the last for p_j provides the expression in (5).

Proof of Corollary 1 Follows by replacing $\mathbf{p}$ in $\mathbf{D}(\mathbf{Q},\mathbf{p})$ with the expression in (4) for (L) and with the one in (5) for (M).

Proof of Proposition 2 Given $p_j^*(\mathbf{Q}|\ \delta(\cdot))$ and $D_j^*(\mathbf{Q}|\ \delta(\cdot))$ defined in Proposition 1 and Corollary 1 for any feasible $\mathbf{Q}$ optimal (which does not guarantee positive profit) best response of downstream firm j should solve the following problem:

$$\max_{Q_j} m_j^*(\mathbf{Q}|\ \delta(\cdot))D_j^*(\mathbf{Q}|\ \delta(\cdot)) - \left(K_j(Q_j) - K_j(Q_j^0)\right)^+ . \tag{.11}$$

Let the best response that solves the problem be $Q_j(\mathbf{Q}_{-j})$. The problem objective in (.11) is not continuously differentiable at Q_j^0 - because of the investment term - and at M^2 because of the premium function and. Two marginal revenues defined (6) and (7) are valid for $Q_j < M^2$ and $Q_j \geq M^2$. Under Assumption 2 (c) and convex increasing investment costs, the objective function is unimodal. Then, we can write the 6 different FOCs for $Q_j(\mathbf{Q}_{-j})$ depending on the 6 possible regions in each of which the objective is continuous and differentiable. $\mathbf{Q}^*(\delta(\cdot))$ becomes the simultaneous solution of a set of FOCs, one for each firm.

Next we show under Assumption 2 (a) and (b) the best responses are non-increasing and result in a unique equilibrium. The slope - first partial derivatives - of the capability best responses:

$$\frac{\partial Q_j(\mathbf{Q}_{-j})}{\partial Q_k} = \begin{cases} -\dfrac{\partial \rho_j^0(\mathbf{Q}|\ \delta(\cdot))/\partial Q_k}{\partial \rho_j^0(\mathbf{Q}|\ \delta(\cdot))/\partial Q_j} & \text{if } Q_j(\mathbf{Q}_{-j}) < Q_j^0, \\ -\dfrac{\partial \rho_j^0(\mathbf{Q}|\ \delta(\cdot))/\partial Q_k}{\partial \rho_j^0(\mathbf{Q}|\ \delta(\cdot))/\partial Q_j - \partial^2 K_j(Q_j)/\partial Q_j^2} & \text{if } Q_j(\mathbf{Q}_{-j}) \in (Q_j^0, M^2), \\ -\dfrac{\partial \rho_j(\mathbf{Q}|\ \delta(\cdot))/\partial Q_k}{\partial \rho_j(\mathbf{Q}|\ \delta(\cdot))/\partial Q_j - \partial^2 K_j(Q_j)/\partial Q_j^2} & \text{if } Q_j(\mathbf{Q}_{-j}) \in (M^2, M^1). \end{cases} \tag{.12}$$

At points Q_j^0, M^2, and M^1, the slope of the best responses are either 0 or one of the neighboring values. Denominators in (.12) are all negative under Assumption 2 (c). Numerators are also negative under Assumption 2 (a). Hence, $\partial Q_j(\mathbf{Q}_{-j})/\partial Q_k$ are non-positive.

The spectral radius of the matrix of the first partials of the capability best responses - denote it with $J(Q)$ - such that $J(Q)_{jj} = 0$ and $J(Q)_{jk} = \dfrac{\partial Q_j(\mathbf{Q}_{-j})}{\partial Q_k}$ for $k \neq j$, is less than 1 if the row sums of the same matrix are strictly higher -1. The sum of the j^{th} row of $J(Q)$ matrix is greater than

$$\begin{array}{ll} -\dfrac{\sum_{k\neq j} \partial \rho_j^0(\mathbf{Q}|\ \delta(\cdot))/\partial Q_k}{\partial \rho_j^0(\mathbf{Q}|\ \delta(\cdot))/\partial Q_j} & \text{if } Q_j(\mathbf{Q}_{-j}) \leq M^2, \\ -\dfrac{\sum_{k\neq j} \partial \rho_j(\mathbf{Q}|\ \delta(\cdot))/\partial Q_k}{\partial \rho_j(\mathbf{Q}|\ \delta(\cdot))/\partial Q_j - \partial^2 K_j(Q_j)/\partial Q_j^2} & \text{if } Q_j(\mathbf{Q}_{-j}) > M^2. \end{array} \tag{.13}$$

which are both strictly greater than -1 under Assumption 2 (a)-(c) . This establishes the uniqueness of $\vec{Q}(\delta(.))$.

Finally, Assumption 2 (c) implies that if $Q_j(\mathbf{Q}_j^0) \geq 0$, it is positive, continuos and decreasing in any Q_k, $k \neq j$. Hence, there is no discontinuity in best responses, which guarantees the existence of a non-negative profit equilibrium for all downstream firms.

Proof of Corollary 3 If $Q_j(0) \leq M^2$, then one of the first 3 cases in Proposition 2 is uniquely satisfied for each downstream firm for $\delta(Q) = 0$, $\forall Q$. $\rho_j^0(\mathbf{Q}|\ \delta(\cdot))$ is decreasing in $\partial\delta(Q)/\partial Q$, $\forall j$.

Proof of Corollary 4 Under symmetry:

(LS)

$$p^*(Q|\ a) = \tilde{\alpha}_1 + (\tilde{\gamma}_{11} + (n-1)\tilde{\gamma}_{12})Q + \tilde{\epsilon}_1 + (\tilde{d}_{11} + (n-1)\tilde{d}_{12})a(Q - M^2),$$

$$D^*(Q|\ a) = \hat{\alpha}_1 + (\hat{\gamma}_{11} + (n-1)\hat{\gamma}_{12})Q + \hat{\epsilon}_1 + (\hat{d}_{11} + (n-1)\hat{d}_{12})a(Q - M^2).$$

$\rho(Q,a) = \partial\left((p^*(Q|\ a) - c_j - c_u - a(Q - M^2))D^*(Q|\ a)\right)/\partial Q_j$ follows from above.

(MS) $m^*(\mathbf{Q}|\ \delta(\cdot))D_j(\mathbf{Q}|\ \delta(\cdot)) = D_j(\mathbf{Q}|\ \delta(\cdot))/\nu_{jj}$. Under symmetry and linear premium function: $\rho(Q,a) = \frac{1}{\nu_{11}}\partial D_j(Q|\ a)/\partial Q_j$.

Proof of Proposition 3 From the Definition in 1, TP is defined by the 3rd case for every downstream firm in Proposition 2. Hence, $\tau = \{Q|\ \rho(Q,0) = 0\}$. Solving this equation for $\rho(Q,a)$ as defined in Corollary 4 for $a = 0$ gives the expressions in (12) and in (13), respectively.

Proof of Proposition 4 Signs of the partials follow from the closed form expressions below.

For the symmetric linear demand case (LS):

$$\frac{\partial\tau}{\partial\alpha_1} = -\beta_{11}(2\beta_{11}\gamma_{11} + \beta_{12}((n-2)\gamma_{11} - (n-1)\gamma_{12}))\Big/$$
$$\left(\kappa_1(2\beta_{11} - \beta_{12})(2\beta_{11} + (n-1)\beta_{12})^2 + \beta_{11}(\gamma_{11} + (n-1)\gamma_{12})(2\beta_{11}\gamma_{11} + \beta_{12}((n-2)\gamma_{11} - (n-1)\gamma_{12}))\right)$$

$$\frac{\partial\tau}{\partial\gamma_{11}} = \beta_{11}\left(\beta_{11}\left(2\beta_{11}\gamma_{11} + \beta_{12}\left(\gamma_{11}(n-2) - \gamma_{12}(n-1)\right)\right)\right.$$
$$\left(\beta_{12}\left(\gamma_{12}\left(n^2 - 4n + 3\right) + 2\gamma_{11}(n-2)\right) + 2\beta_{11}\left(2\gamma_{11} + \gamma_{12}(n-1)\right)\right)$$
$$-\left(2\beta_{11} + \beta_{12}(n-2)\right)\left((2\beta_{11} - \beta_{12})\kappa_1\left(2\beta_{11} + \beta_{12}(n-1)\right)^2 + \beta_{11}\left(\gamma_{11} + \gamma_{12}(n-1)\right)\right.$$
$$\left.\left.\left(2\beta_{11}\gamma_{11} + \beta_{12}\left(\gamma_{11}(n-2) - \gamma_{12}(n-1)\right)\right)\right)\right)$$
$$\left(\alpha_1 + (c_u + c_1)\left(\beta_{11} + \beta_{12}(n-1)\right)\right)\Big/$$
$$\left((2\beta_{11} - \beta_{12})\kappa_1\left(2\beta_{11} + \beta_{12}(n-1)\right)^2 + \beta_{11}\left(\gamma_{11} + \gamma_{12}(n-1)\right)\left(2\beta_{11}\gamma_{11} + \beta_{12}\left(\gamma_{11}(m-2) - \gamma_{12}(n-1)\right)\right)\right)^2$$

$$\frac{\partial \tau}{\partial \beta_{12}} =$$

$$\beta_{11} \left(\left(2\beta_{11}\gamma_{11} + \beta_{12} \left(\gamma_{11}(n-2) - \gamma_{12}(n-1) \right) \right) \right.$$

$$\left(-\kappa_1 \left(2\beta_{11} + \beta_{12}(n-1) \right)^2 + 2 \left(2\beta_{11} - \beta_{12} \right) \kappa_1 (n-1) \left(2\beta_{11} + \beta_{12}(n-1) \right) \right.$$

$$\left. + \; \beta_{11} \left(\gamma_{11} + \gamma_{12}(n-1) \right) \left(\gamma_{11}(n-2) - \gamma_{12}(n-1) \right) \right)$$

$$\left(\alpha_1 + (c_u + c_1) \left(\beta_{11} + \beta_{12}(n-1) \right) \right) - \left(\gamma_{11}(n-2) - \gamma_{12}(n-1) \right)$$

$$\left((2\beta_{11} - \beta_{12}) \kappa_1 \left(2\beta_{11} + \beta_{12}(n-1) \right)^2 + \beta_{11} \left(\gamma_{11} + \gamma_{12}(n-1) \right) \left(2\beta_{11}\gamma_{11} + \beta_{12} \left(\gamma_{11}(n-2) - \gamma_{12}(n-1) \right) \right) \right)$$

$$\left(\alpha_1 + (c_u + c_1) \left(\beta_{11} + \beta_{12}(n-1) \right) \right) - (n-1) (c_u + c_1) \left(2\beta_{11}\gamma_{11} + \beta_{12} \left(\gamma_{11}(n-2) - \gamma_{12}(n-1) \right) \right)$$

$$\left. \left((2\beta_{11} - \beta_{12}) \kappa_1 \left(2\beta_{11} + \beta_{12}(n-1) \right)^2 + \beta_{11} \left(\gamma_{11} + \gamma_{12}(n-1) \right) \left(2\beta_{11}\gamma_{11} + \beta_{12} \left(\gamma_{11}(n-2) - \gamma_{12}(n-1) \right) \right) \right) \right)$$

$$\Big/ \left((2\beta_{11} - \beta_{12}) k_1 \left(2\beta_{11} + \beta_{12}(n-1) \right)^2 + \beta_{11} \left(\gamma_{11} + \gamma_{12}(n-1) \right) \left(2\beta_{11}\gamma_{11} + \beta_{12} \left(\gamma_{11}(n-2) - \gamma_{12}(n-1) \right) \right) \right)^2$$

$$\frac{\partial \tau}{\partial \gamma_{12}} = \beta_{11} \left(\beta_{11}(n-1) \left(2\beta_{11}\gamma_{11} + \beta_{12} \left(\gamma_{11}(n-3) - 2\gamma_{12}(n-1) \right) \right) \right.$$

$$\left(2\beta_{11}\gamma_{11} + \beta_{12} \left(\gamma_{11}(n-2) - \gamma_{12}(n-1) \right) \right) - \beta_{12}(1-n) \left((2\beta_{11} \right.$$

$$\left. \left. -\beta_{12}) \kappa_1 \left(2\beta_{11} + \beta_{12}(n-1) \right)^2 + \beta_{11} \left(\gamma_{11} + \gamma_{12}(n-1) \right) \left(2\beta_{11}\gamma_{11} + \beta_{12} \left(\gamma_{11}(n-2) - \gamma_{12}(n-1) \right) \right) \right) \right)$$

$$\left(\alpha_1 + (c_u + c_1) \left(\beta_{11} + \beta_{12}(n-1) \right) \right)$$

$$\Big/ \left((2\beta_{11} - \beta_{12}) k_1 \left(2\beta_{11} + \beta_{12}(n-1) \right)^2 + \beta_{11} \left(\gamma_{11} + \gamma_{12}(n-1) \right) \left(2\beta_{11}\gamma_{11} + \beta_{12} \left(\gamma_{11}(n-2) - \gamma_{12}(n-1) \right) \right) \right)^2$$

$$\frac{\partial \varpi}{\partial \beta_{11}} =$$

$$- \frac{2\beta_{11}\gamma_{11} \left(\alpha_1 + (c_u + c_1) \left(\beta_{11} + \beta_{12}(n-1) \right) \right)}{(2\beta_{11} - \beta_{12}) k_1 \left(2\beta_{11} + \beta_{12}(n-1) \right)^2 + \beta_{11} \left(\gamma_{11} + \gamma_{12}(n-1) \right) \left(2\beta_{11}\gamma_{11} + \beta_{12} \left(\gamma_{11}(n-2) - \gamma_{12}(n-1) \right) \right)}$$

$$- \frac{\left(2\beta_{11}\gamma_{11} + \beta_{12} \left(\gamma_{11}(n-2) - \gamma_{12}(n-1) \right) \right) \left(\alpha_1 + (c_u + c_1) \left(\beta_{11} + \beta_{12}(n-1) \right) \right)}{(2\beta_{11} - \beta_{12}) \kappa_1 \left(2\beta_{11} + \beta_{12}(n-1) \right)^2 + \beta_{11} \left(\gamma_{11} + \gamma_{12}(n-1) \right) \left(2\beta_{11}\gamma_{11} + \beta_{12} \left(\gamma_{11}(n-2) - \gamma_{12}(n-1) \right) \right)}$$

$$+ \left(\beta_{11} \left(2\beta_{11}\gamma_{11} + \beta_{12} \left(\gamma_{11}(n-2) - \gamma_{12}(n-1) \right) \right) \left(2\kappa_1 \left(2\beta_{11} + \beta_{12}(n-1) \right)^2 \right. \right.$$

$$+ 4 \left(2\beta_{11} - \beta_{12} \right) k_1 \left(2\beta_{11} + \beta_{12}(n-1) \right) + 2\beta_{11}\gamma_{11} \left(\gamma_{11} + \gamma_{12}(n-1) \right) + \left(\gamma_{11} + \gamma_{12}(n-1) \right)$$

$$\left. \left(2\beta_{11}\gamma_{11} + \beta_{12} \left(\gamma_{11}(n-2) - \gamma_{12}(n-1) \right) \right) \right)$$

$$\left. \left(\alpha_1 + (c_u + c_1) \left(\beta_{11} + \beta_{12}(n-1) \right) \right) \right) \Big/$$

$$\left((2\beta_{11} - \beta_{12}) \kappa_1 \left(2\beta_{11} + \beta_{12}(n-1) \right)^2 + \beta_{11} \left(\gamma_{11} + \gamma_{12}(n-1) \right) \left(2\beta_{11}\gamma_{11} + \beta_{12} \left(\gamma_{11}(n-2) - \gamma_{12}(n-1) \right) \right) \right)^2$$

$$- \frac{\beta_{11} (c_u + c_1) \left(2\beta_{11}\gamma_{11} + \beta_{12} \left(\gamma_{11}(n-2) - \gamma_{12}(n-1) \right) \right)}{(2\beta_{11} - \beta_{12}) \kappa_1 \left(2\beta_{11} + \beta_{12}(n-1) \right)^2 + \beta_{11} \left(\gamma_{11} + \gamma_{12}(n-1) \right) \left(2\beta_{11}\gamma_{11} + \beta_{12} \left(\gamma_{11}(n-2) - \gamma_{12}(n-1) \right) \right)}$$

$$\frac{\partial\tau}{\partial\kappa_1} =$$

$$\beta_{11}\left(2\beta_{11}-\beta_{12}\right)\left(2\beta_{11}+\beta_{12}(n-1)\right)^2\left(2\beta_{11}\gamma_{11}+\beta_{12}\left(\gamma_{11}(n-2)-\gamma_{12}(n-1)\right)\right)$$
$$\left(\alpha_1+\left(c_u+c_1\right)\left(\beta_{11}+\beta_{12}(n-1)\right)\right)\Big/$$
$$\left(\left(2\beta_{11}-\beta_{12}\right)\kappa_1\left(2\beta_{11}+\beta_{12}(n-1)\right)^2+\beta_{11}\left(\gamma_{11}+\gamma_{12}(n-1)\right)\left(2\beta_{11}\gamma_{11}+\beta_{12}\left(\gamma_{11}(n-2)-\gamma_{12}(n-1)\right)\right)\right)^2$$

For the symmetric multiplicative demand case (MS):

$$\frac{\partial\tau}{\partial\lambda_1}=\frac{1}{\lambda_1\left(\vartheta_{11}+k_1+\vartheta_{12}(n-1)\right)}$$

$$\frac{\partial\tau}{\partial\vartheta_{11}}=\left(-\log\left(\frac{\lambda_1\vartheta_{11}}{\nu_{11}}\right)-(n-1)\log\left(1-e^{\beta_{12}\left(-\frac{1}{\nu_{11}}-c_u-c_1\right)}\right)\right.$$
$$\left.+\ \nu_{11}\left(c_u+c_1\right)+\log\left(g_1\right)+\frac{\gamma_{11}+k_1+\vartheta_{12}(m-1)}{\vartheta_{11}}+\log\left(k_1\right)+1\right)\Big/\left(\vartheta_{11}+k_1+\vartheta_{12}(n-1)\right)^2$$

$$\frac{\partial\tau}{\partial\nu_{12}}=\frac{(n-1)\left(\nu_{11}c_1+\nu_{11}c_u+1\right)}{\nu_{11}\left(e^{\beta_{12}\left(\frac{1}{\nu_{11}}+c_u+c_1\right)}-1\right)\left(\vartheta_{11}+k_1+\vartheta_{12}(n-1)\right)},$$

$$\frac{\partial\tau}{\partial\vartheta_{12}}=-\frac{(n-1)\left(\log\left(\frac{\lambda_1\vartheta_{11}}{\nu_{11}}\right)+(n-1)\log\left(1-e^{\beta_{12}\left(-\frac{1}{\nu_{11}}-c_u-c_1\right)}\right)-\nu_{11}\left(c_u+c_1\right)-\log\left(g_1\right)-\log\left(k_1\right)-1\right)}{\left(\vartheta_{11}+k_1+\vartheta_{12}(n-1)\right)^2},$$

$$\frac{\partial\tau}{\partial\nu_{11}}=\left(-\log\left(\frac{\lambda_1\vartheta_{11}}{\nu_{11}}\right)-(n-1)\log\left(1-e^{\nu_{12}\left(-\frac{1}{\nu_{11}}-c_u-c_1\right)}\right)\right.$$
$$\left.+\ \nu_{11}\left(c_u+c_1\right)+\log\left(g_1\right)+\frac{\vartheta_{11}+k_1+\vartheta_{12}(n-1)}{\vartheta_{11}}+\log\left(k_1\right)+1\right)\Big/\left(\vartheta_{11}+k_1+\vartheta_{12}(n-1)\right)^2,$$

$$\frac{\partial\tau}{\partial k_1}=\left(-\log\left(\frac{\lambda_1\vartheta_{11}}{\nu_{11}}\right)-(n-1)\log\left(1-e^{\nu_{12}\left(-\frac{1}{\nu_{11}}-c_u-c_1\right)}\right)\right.$$
$$\left.+\nu_{11}\left(c_u+c_1\right)+\log\left(g_1\right)-\frac{\vartheta_{11}+k_1+\vartheta_{12}(n-1)}{k_1}+\log\left(k_1\right)+1\right)\Big/\left(\vartheta_{11}+k_1+\vartheta_{12}(n-1)\right)^2$$

C. Auxiliary Material for Symmetric Downstream Tier

C.1. Approximating Upstream Return

The maximum net contribution an upstream leader can receive from a capability level of M^1 - $M^1 \in [M^2,\tau]$ - from a symmetric tier of n firms can be described with the following mathematical problem where

$Q'(a)$ is the symmetric unconstrained downstream capability equilibrium, i.e.,

$$Q'(a) = \left\{ Q \middle| \; \rho^0(Q|\, a) = \frac{\partial K(Q)}{\partial Q} \vee \rho^1(Q|\, a) = \frac{\partial K(Q)}{\partial Q} \right\}.$$

Only one of the two conditions hold depending on the value of a. For small values of a, second one is likely to hold.

$$R(M^1) = \max_a \; n \, a(\underline{\mathrm{M}}^1 - M^2) D^*(\underline{\mathrm{M}}^1|\, a) \tag{.14}$$

$$s.t: \underline{\mathrm{M}}^1 \leq M^1, \tag{.15}$$

$$Q'(a) \geq \underline{\mathrm{M}}^1. \tag{.16}$$

where $\underline{\mathrm{M}}^1$ is the capability level offered and adopted. (Constraint (.16) ensures that this is the case.)

We define two pseudo revenue functions: $R''(M^1)$ is the upstream revenue when it induces the adoption of all of its capability. (Constraint (.15) is made to hold strictly.) $R'(M^1)$ is the upstream revenue when it induces the adoption of all of its capability and charges the highest premium possible. (Constraints (.15) and (.16) are made to hold strictly.) The two approximations can be written as the solution of the following

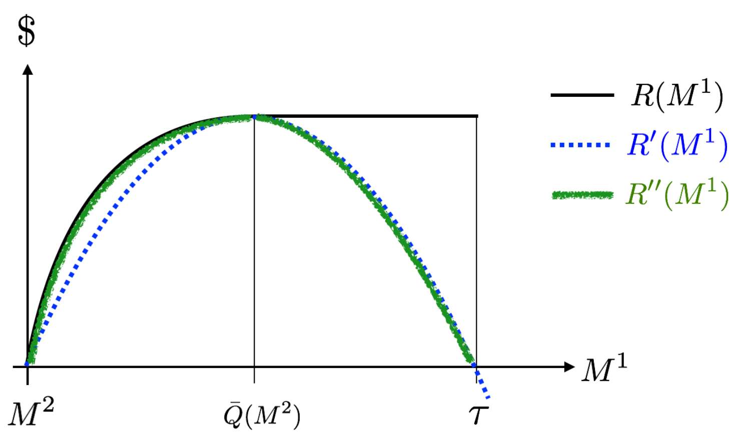


Figure .2: Approximating $R(M^1)$ at $\bar{Q}(M^2)$ using two functions: $R''(M^1)$ and $R'(M^1)$.

problems.

$$R''(M^1) = \max_a \; n\, a(\underline{\mathrm{M}}^1 - M^2)D^*(\underline{\mathrm{M}}^1 |\, a) \tag{.17}$$

$$s.t : \underline{\mathrm{M}}^1 = M^1, \tag{.18}$$

$$Q'(a) \geq \underline{\mathrm{M}}^1. \tag{.19}$$

$$R'(M^1) = \max_a \; n\, a(\underline{\mathrm{M}}^1 - M^2)D^*(\underline{\mathrm{M}}^1 |\, a) \tag{.20}$$

$$s.t : \underline{\mathrm{M}}^1 = M^1, \tag{.21}$$

$$Q'(a) = \underline{\mathrm{M}}^1. \tag{.22}$$

Given the identical objective functions and tighter constraints: $R(M^1) \geq R''(M^1) \geq R'(M^1)$, for all $M^{1\text{`}} \in [M^1, \tau]$.

Lemma 1. *$R(M^1)$ defined as the solution of (.14) - (.16) is quasi-linear and non-decreasing in M^1.*

Proof Let the feasible region of the problem in (.14) - (.16) for any M^1 be denoted by $\mathbb{X}(M^1)$. $\underline{\mathrm{M}}^1 \leq M^1$. Hence, for any M^a and M^b such that $M^a < M^b$, $\mathbb{X}(M^b) \supseteq \mathbb{X}(M^a)$. Hence, the optimal value of the objective is non-decreasing in M^1: $R(M^b) \geq R(M^a)$.

Lemma 2. *For any $M^2 \leq \tau$, there exists a capability level - denote it with $\bar{Q}(M^2)$ - such that $R(M^1) = R(\bar{Q}(M^2))$ for all $M^1 \geq \bar{Q}(M^2)$. Moreover $\bar{Q}(M^2) < \tau$.*

Proof Define $\underline{\mathrm{M}}^{1*}(M^1)$ as the optimal value for the problem (.14)-(.16). We need to show that the constraint (.15) is not binding for $M^1 \geq \bar{Q}(M^2)$ for a threshold $\bar{Q}(M^2)$ strictly less than τ.

Suppose otherwise. Then for some M^2, $\underline{\mathrm{M}}^{1*}(\tau) = \tau$. But, $Q'(a) \geq \tau \Rightarrow a^* = 0 \wedge R(M^1) = 0$. There exists $\epsilon \geq 0$ such that $a'(\tau - \epsilon) > 0$ and $D^*(\tau - \epsilon |\, a'(\tau - \epsilon)) > 0$. Contradiction for optimality.

Lemma 2 describes the maximum technology that can pass from the upper tier to the lower tier as a function of the upstream laggard capability level. It also shows that it is strictly less than τ. It has the following corollary result.

Corollary 1.

1. *For all $M^2 \leq \tau$, $\max_{M^1 \in [M^2, \tau]} R(M^1) = R(\bar{Q}(M^2))$.*
2. $\arg\max_{M^1} R(M^1) \supseteq [\bar{Q}(M^2), \tau]$.
3. $R''(\bar{Q}(M^2)) = R(\bar{Q}(M^2))$.

Proof Result (1) and (2) follow directly from Lemma 1 and Lemma 2. Result (3) follow from the fact that Problems (.14)-(.16) and (.17) -(.19) become identical for $M^1 = \bar{Q}(M^2)$ since at optimality Constraint (.15) is binding.

Lemma 3. *$R''(M^1) = R'(M^1)$ for all $M^1 \geq \bar{Q}(M^2)$.*

Proof Results from the fact that for $M^1 \geq \bar{Q}(M^2)$, Constraint (.19) is binding. Suppose it were not. Note that for both problems that define $R''(M^1)$ and $R'(M^1)$, $\underline{M}^1 = M^1$. Then in the optimal solution of (.17) -(.19) $a^*(M^1) < a'(M^1)$. Then, $\exists\ M^x > M^1$ such that $a^*(M^1) = a'(M^x)$ as $a'(M^1)$ is strictly decreasing in M^1. But, then the objective in (.17) can attain a higher value at $na^*(M^1)(M^x - M^2)D^*(M^x|\ a^*(M^1))$. Contradiction.

Corollary 2. *$R'(\bar{Q}(M^2)) = R(\bar{Q}(M^2))$.*

Proof Follows from Lemma 3 and Corollary 1.

C.2. Fixed Technological Potential Numerical Analysis (For §5.2)

Table .2: Δ ratios in the (LS) model

Factor pairs (LS)	Q^*	D^*	p^*	δ^*	R^*	π^*
(α_1, γ_{12})	-0.0043	0.0407	0.0951	0.3567	0.6016	0.4981
(α_1, β_{11})	0.0010	0.0421	0.0291	0.1683	0.3364	0.2812
(α_1, κ_1)	0.0069	0.0378	0.0902	0.6613	0.8828	0.4317
$(\gamma_{11}, \gamma_{12})$	0.1313	-0.6234	-0.2034	1.1390	0.2165	-2.0231
$(\gamma_{11}, \beta_{11})$	-0.0087	0.1046	-0.5850	-0.7155	-0.6758	-0.8599
(γ_{11}, κ_1)	-0.0274	0.6625	0.4851	0.0600	0.7321	1.2316
$(\beta_{12}, \gamma_{12})$	0.0104	0.0008	0.0566	0.2594	0.2606	0.0843
(β_{12}, β_{11})	-0.0323	0.8737	-0.1314	-0.3736	0.2836	3.9794
(β_{12}, κ_1)	-0.0368	0.9533	0.7789	0.3730	1.4422	4.9132

Table .3: Equilibrium of the (MS) base case under the modification of different tuples for fixed TP

Modification on Base Case	Q^*	D^*	p^*	δ^*	R^*	π^*
$(\lambda_1^l, \vartheta_{12}^l) = (491.3774, 0.01)$	9.1308	2.1567	1.7467	0.1967	2.1211	2.6901
$(\lambda_1^h, \vartheta_{12}^h) = (48585.0, 0.10)$	6.7408	1767.0134	1.7027	0.1527	1348.9679	2208.7627
$(\lambda_1^l, \nu_{11}^l) = (68.6406, 0.10)$	7.3664	3.7475	11.7066	1.4066	26.3568	37.4706
$(\lambda_1^h, \nu_{11}^h) = (25230.359, 1.3)$	7.5955	61.7278	1.1811	0.1119	34.5303	47.4783
$(\lambda_1^l, k_1^l) = (158.5617, 0.01)$	7.5775	2.6944	1.7313	0.1813	2.4428	3.3672
$(\lambda_1^h, k_1^h) = (35825.3808, 0.10)$	7.5787	267.8892	1.7314	0.1814	243.0237	334.8501
$(\lambda_1^l, g_1^l) = (500.0, 0.001)$	7.5783	3.7390	1.7314	0.1814	3.3912	4.6733
$(\lambda_1^h, g_1^h) = (50000.0, 0.10)$	7.5783	373.8984	1.7314	0.1814	339.1172	467.3269
$(\vartheta_{11}^l, \vartheta_{12}^l) = (0.3857, 0.01)$	7.3524	143.4139	1.6207	0.0707	50.6702	179.2630
$(\vartheta_{11}^h, \vartheta_{12}^h) = (0.0877, 0.0700)$	7.8722	12.7287	1.8664	0.3164	20.1397	15.9061
$(\vartheta_{11}^l, \nu_{11}^l) = (0.3947, 0.10)$	6.6573	413.0920	10.8095	0.5095	1052.3510	4130.9158
$(\vartheta_{11}^h, \nu_{11}^h) = (0.1636, 1.00)$	7.8664	20.4767	1.4788	0.1788	18.3065	20.4719
$(\vartheta_{11}^l, k_1^l) = (0.3217, 0.01)$	6.9504	46.1582	1.6439	0.0939	21.6646	57.6970
$(\vartheta_{11}^h, k_1^h) = (0.0927, 0.10)$	8.5667	23.0530	1.8905	0.3405	39.2426	28.8027
$(\vartheta_{11}^l, g_1^l) = (0.3070, 0.001)$	7.0095	45.3510	1.6514	0.1014	23.0019	56.6883
$(\vartheta_{11}^h, g_1^h) = (0.0668, 0.100)$	8.9025	17.9576	1.9493	0.3993	35.8553	22.3910
$(\nu_{12}^l, \vartheta_{12}^l) = (0.15880.01)$	9.1387	2.4764	1.7469	0.1969	2.4381	3.0897
$(\nu_{12}^h, \vartheta_{12}^h) = (0.9812, 0.06)$	7.2862	74.4147	1.7235	0.1735	64.5498	93.0140
$(\nu_{12}^l, \nu_{11}^l) = (0.0406, 0.10)$	7.6903	5.4043	11.7715	1.4715	39.7630	54.0384
$(\nu_{12}^h, \nu_{11}^h) = (0.8072, 1.00)$	7.5565	45.5459	1.4447	0.1447	32.9517	45.5413
$(\nu_{12}^l, k_1^l) = (0.2429, 0.0100)$	7.6543	2.9511	1.7331	0.1831	2.7018	3.6881
$(\nu_{12}^h, k_1^h) = (3.0083, 0.10)$	7.3758	216.8287	1.7261	0.1761	190.9524	271.0250
$(\nu_{12}^l, g_1^l) = (0.2685, 0.0010)$	7.6491	4.0653	1.7330	0.1830	3.7205	5.0812
$(\nu_{12}^h, g_1^h) = (2.7011, 0.0700)$	7.3817	213.2620	1.7263	0.1763	187.9550	266.5470

Table .4: Δ ratios in the (MS) model

Factor pairs (MS)	Q^*	D^*	p^*	δ^*	R^*	π^*
$(\lambda_1, \vartheta_{12})$	-0.27	8.3608	-0.0003	-0.0023	6.4875	8.3788
(λ_1, ν_{11})	0.0001	0.0422	-0.0025	-0.0025	0.0008	0.0007
(λ_1, k_1)	0	0.4376	0	0	0.4378	0.4376
(λ_1, g_1)	0	1	0	0	1	1
$(\vartheta_{11}, \vartheta_{12})$	-0.0997	1.2849	-0.2138	-4.9044	0.8496	1.2849
$(\vartheta_{11}, \nu_{11})$	0.1220	-0.6384	-0.5798	-0.4360	-0.6600	-0.6683
(ϑ_{11}, k_1)	-0.4035	0.8686	-0.2603	-4.5583	-1.4079	0.8690
(ϑ_{11}, g_1)	-0.4440	0.9931	-0.2966	-4.8280	-0.9187	0.9947
$(\nu_{12}, \vartheta_{12})$	-0.0391	5.6082	-0.0026	-0.0230	4.9183	5.6188
(ν_{12}, ν_{11})	-0.0016	0.6809	-0.0804	-0.0827	-0.0157	-0.0144
(ν_{12}, k_1)	-0.0032	6.3664	-0.0004	-0.0033	6.1207	6.3675
(ν_{12}, g_1)	-0.0039	5.6784	-0.0004	-0.0041	5.4643	5.6782

Table .5: Equilibrium of the (LS) base case under the modification of different tuples for fixed TP

Modification on Base Case	Q^*	D^*	p^*	δ^*	R^*	π^*
$(\alpha_1^l,\ \gamma_{12}^l) = (19.549, -0.15)$	73.0388	111.9471	193.5856	8.3403	3734.6834	7367.4160
$(\alpha_1^h,\ \gamma_{12}^h) = (294.59, -0.8)$	68.6449	176.0367	452.5459	50.1912	35341.9847	58996.0950
$(\alpha_1^l,\ \beta_{11}^l) = (11.20, -0.65)$	72.3613	104.0770	185.0831	6.1661	2567.0162	5499.5166
$(\alpha_1^h,\ \beta_{11}^h) = (210.5603, -1.0)$	73.6882	181.9722	280.6947	24.6056	17910.1652	32971.6373
$(\alpha_1^l, \kappa_1^l) = (5.9632, 2.10)$	71.4914	95.0530	148.6095	3.1299	1190.0151	3066.6078
$(\alpha_1^h, \kappa_1^h) = (52.7638, 2.65)$	75.1300	121.4614	247.2391	18.3557	8918.0330	12806.0963
$(\gamma_{11}^l, \gamma_{12}^l) = (1.9196, -0.1)$	74.0542	122.5575	230.1185	13.6841	6708.3423	12778.8521
$(\gamma_{11}^h, \gamma_{12}^h) = (2.6858, -0.5)$	78.1513	90.3705	210.4021	20.2503	7320.1350	1887.9365
$(\gamma_{11}^l, \beta_{11}^l) = (1.4914, -0.5)$	74.9680	113.9197	332.9445	26.2013	11939.3703	20859.4416
$(\gamma_{11}^h, \beta_{11}^h) = (2.7616, -1.0)$	74.4055	124.2530	164.3299	9.9442	4942.3992	5305.1688
$(\gamma_{11}^l, \kappa_1^l) = (1.8982, 2.0)$	75.2106	101.3502	205.1765	15.0226	6090.1878	7928.4556
$(\gamma_{11}^h, \kappa_1^h) = (2.1565, 2.65)$	74.7180	117.4016	228.9695	15.2382	7155.9734	10262.8137
$(\beta_{12}^l, \gamma_{12}^l) = (0.1156, 0.0)$	73.7144	114.0755	208.1552	11.2225	5120.8690	8846.4616
$(\beta_{12}^h, \gamma_{12}^h) = (0.2217, -0.25)$	75.1634	114.2503	230.4253	16.7276	7644.5508	10256.9117
$(\beta_{12}^l, \beta_{11}^l) = (0.0762, -0.50)$	76.1448	81.3997	238.6590	18.8899	6150.5295	3369.6529
$(\beta_{12}^h, \beta_{11}^h) = (0.0762, -0.50)$	74.5153	128.5810	217.8602	14.2085	7307.7649	12265.8577
$(\beta_{12}^l, \kappa_1^l) = (0.1427, 2.0)$	75.5792	91.6824	186.9860	13.9278	5107.7323	4414.4513
$(\beta_{12}^h, \kappa_1^h) = (0.2127, 2.65)$	74.6740	120.1659	234.4492	15.6209	7508.3955	11482.7103