# Online Appendix for When Consumers Anticipate Liquidation Sales: Managing Operations under Financial Distress

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## Appendix D: Proofs of Supplemental Results in Appendix C

*Proof of Lemma C.1.* We consider two scenarios: when customers purchase in the first period and when customers wait in the first period. The scenario where customers purchase in the first period is exactly the same as in the inventory-clearance case. Therefore, we focus on the case where strategic customers wait. Under the objective of revenue maximization, the second-period price is $s$ when the firm does not go bankrupt. In bankruptcy, with waiting strategic customers, if the firm sets the price at $s$, its revenue is $s\min(q-(1-\alpha)D, \alpha D)$, and if it sets the price at $b$, its revenue is $b[q-(1-\alpha)D]^+$. If $q-(1-\alpha)D < \alpha D$, setting the price at $s$ is clearly better. Otherwise, setting the price at $s$ is better if and only if $\alpha Ds > b(q-(1-\alpha)d)$, i.e. $D < \frac{bq}{\alpha s+(1-\alpha)b}$. $\square$

*Proof of Lemma C.2.* The profit function in the buy-region, i.e. when $(p,q)\in\Omega^B$, is discussed in Section 5.1 and is omitted here.

Within the wait-region, i.e. $q > Q_b(p)$, the firm effectively faces first-period demand $(1-\alpha)D$ as only myopic customers make purchases. We consider two further scenarios.

1. When $\tau \le [(1-\alpha)p-c]q$, that is, $q \ge d_\tau^W$, the firm's profit depends on the realized demand $D$ according to the following three cases.

   (a) for $D \ge \frac{q}{1-\alpha}$, the firm does not go bankrupt and has no leftover inventory to salvage in the second period; thus, $R_1 = pq$ and $R_2 = 0$.

   (b) for $D \in \left[d_\tau^W, \frac{q}{1-\alpha}\right)$, the firm does not go bankrupt. However, it does have leftover inventory $q-(1-\alpha)D$ and this inventory is liquidated at price $s$. Therefore, $R_1=(1-\alpha)pD$ and $R_2 = s[q-(1-\alpha)D]$.

   (c) for $D < d_\tau^W$, the firm goes bankrupt and the leftover inventory $q-(1-\alpha)D$ is liquidated at price $b$, leading to $R_1 = (1-\alpha)pD$ and $R_2 = b[q-(1-\alpha)D]$.

Integrating $D$ over the above three scenarios leads to (24).

2. When $\tau > [(1-\alpha)p-c]q$, i.e. $q < d_\tau^W$, we can discuss the firm's profit according to three similar scenarios, leading to (25). $\square$

*Proof of Lemma C.3.* First, we first show that $p^{W,*}=v$ by contradiction. Assume $(p,q)\in\Omega^W$ with $p<v$ is globally optimal. Now consider solution $(p',q)\in\Omega_0$ with $p'\in(p,v]$. Consider two scenarios.

1. When $(p',q)\in\Omega^B$, for any demand realization $D\in[d_l,d_h]$, we have $\min(q,D)\ge\min(q,(1-\alpha)D)$. Therefore, the firm sells more units under $(p',q)$ at a higher price than under $(p,q)$. Hence, the firm's first-period revenue $R_1$ is higher under $(p',q)$, leading to a lower probability of liquidation sale, and the average second-period price is therefore also higher under $(p',q)$. Therefore, $\pi(p',q) > \pi(p,q)$.

2. When $(p',q) \in \Omega^W$. As myopic customers purchase as long as $p' \le v$ and strategic customers do not purchase, the firm sells the same unit $\min(q,(1-\alpha)D)$ under $(p',q)$ as under $(p,q)$ and, hence, has a higher $R_1$ due to the higher price. In the second period, it has the same amount of leftover inventory $[q-(1-\alpha)D]^+$ under $(p',q)$ as under $(p',q)$ and a higher expected second-period price due to the lower probability of bankruptcy. Therefore, $\pi(p',q) > \pi(p,q)$.

Combining the two scenarios, we have that the profit under $(p',q)$ is greater than $(p,q)$, contradicting the assumption that $(p,q)$ is globally optimal. Therefore, $p=v$ is necessary for $(p,q)\in\Omega^W$ to be globally optimal. In other words, $p^{W,*}=v$.

Given the above results, when identifying $q^{W,*}$, we can simplify the profit function to $\pi_L^W(v,q)$ and $\pi_H^W(v,q)$. In the following, we first examine the (pseudo-)concavity of $\pi_L^W(v,q)$ and $\pi_H^W(v,q)$, which allows us to use the first-order condition only to identify the global optima. For ease of exposition, we suppress $v$ in the profit function for the rest of this proof.

We first show that $\pi_H^W(q)$ is pseudo-concave. Taking derivatives of $\pi_H^W$ with respect to $q$, we have:

$$\frac{d\pi_H^W}{dq} = (v-s)\bar{F}\left(\frac{q}{1-\alpha}\right) + (s-b)\bar{F}(q)\left[1-\alpha q h(q)\right] - (c-b). \tag{27}$$

At $\frac{d\pi_H^W}{dq}=0$, we have:

$$\begin{aligned}\frac{d^2\pi_H^W}{dq^2} &= -\left(\frac{v-s}{1-\alpha}\right) f\left(\frac{q}{1-\alpha}\right) - (s-b)\bar{F}(q)[(2-\alpha q h(q))h(q) + q h'(q)] \\ &\le -\left(\frac{v-s}{1-\alpha}\right) f\left(\frac{q}{1-\alpha}\right) - (s-b)\bar{F}(q)\left[\left(1+\frac{(c-b)-(v-s)\bar{F}\left(\frac{q}{1-\alpha}\right)}{(s-b)\bar{F}(q)}\right)h(q)\right] \\ &= -\left(\frac{v-s}{1-\alpha}\right)\bar{F}\left(\frac{q}{1-\alpha}\right)\left[h\left(\frac{q}{1-\alpha}\right) - (1-\alpha)h(q)\right] - [(s-b)\bar{F}(q)+(c-b)]h(q) < 0.\end{aligned} \tag{28}$$

Therefore, $\pi_H^W(q)$ is psuedo-concave in $q$. Setting $\frac{d\pi_H^W}{dq}=0$ leads to $q_H^W$.

Second, we show that $\pi_L^W(q)$ is pseudo-concave for $[(1-\alpha)v-c]q \ge \tau$, i.e. the region where $\pi_L^W(q)$ is the relevant profit function. Define $k(q)$ as follows:

$$k(q) = \left(1-\frac{c}{v}\right)d_\tau^W - \frac{\tau}{(1-\alpha)v} = \frac{c}{(1-\alpha)v^2}[(v-c)q-\tau]. \tag{29}$$

It is easy to check that $k(q)>0$ for $\tau \le (v-c)q$. Taking derivatives, we have

$$\frac{d\pi_L^W}{dq} = (v-s)\bar{F}\left(\frac{q}{1-\alpha}\right) + (s-b)\bar{F}\left(d_\tau^W\right)\left[1-k(q)h(d_\tau^W)\right] - (c-b). \tag{30}$$

Similarly to the case of $\pi_H^W$, we can also show that $\frac{d^2\pi_L^W(v,q)}{dq^2}<0$ at $\frac{d\pi_L^W}{dq}=0$ when $\tau\le(v-c)q$. Setting $\frac{d\pi_H^W}{dq}=0$ leads to $q_L^W$ as in (4).

In addition, note that $\pi_L^W$ is relevant when $[(1-\alpha)v-c]q\ge\tau$ and $\pi_H^W$ is relevant otherwise; at $q=\frac{\tau}{(1-\alpha)v-c}$ (assuming this value is in the region of $(d_l,(1-\alpha)q^{NV}]$),

$$\frac{d\pi_H^W}{dq} - \frac{d\pi_L^W}{dq} = (s-b)f(q)[k(q)-\alpha q] = (s-b)f(q)\left(\frac{[(1-\alpha)v-c]q}{v} - \frac{\tau}{(1-\alpha)v}\right) < 0. \tag{31}$$

Therefore, $q=\frac{\tau}{(1-\alpha)v-c}$ cannot be a local optima, and hence, within the wait-region, only $q_L^W$ or $q_H^W$ as defined above can be the global optima. □

*Proof of Corollary C.1.* First, note that by definition, $q_L^W$ and $q_H^W$ are smaller than $(1-\alpha)q^{NV}$. For $\tau \le (1-\alpha)(vd_l - cq^{NV})$, $\exists\ q \ge (1-\alpha)q^{NV}$ such that $(v,\, q) \in \Omega^B$. Therefore, we have $\Pi_L^B(v,q) > \Pi_L^B(v, q_i^W) > \Pi_i^W(v,\, q_i^W)$ for $i = H, L$. Therefore, neither $(v, q_L^W)$ nor $(v, q_H^W)$ can be the global optima.

Combining this with Corollary (2), we can immediately get the first result.

To see the second result, note that when $d_l > (1-\alpha)q^{NV}$, the solution with $(p,\, q) = (v,\, d_l)$, which is in the buy-region, dominates $(v, q_L^W)$ and $(v, q_H^W)$. □

*Proof of Corollary C.2.* Note that $q_L^W$ maximizes $\pi_L^W(v,q)$, subject to $[(1-\alpha)v - c]q \ge \tau$. As the feasible set decreases as $\tau$ increases, $\pi_L^W(v, q_l^W)$ decreases in $\tau$. On the other hand, $\pi_H^W(v, q_h^W)$ is independent of $\tau$, while the feasible set expands as $\tau$ increases. Therefore, $\pi(v, q_L^W) - \pi_(v, q_H^W)$ decreases in $\tau$. In addition, note that $q_L^W$ becomes irrelevant when $\tau$ is sufficiently large and $q_H^W$ becomes irrelevant when $\tau$ is sufficiently small. Therefore, there exists a threshold $T^W(\alpha)$ such that $\pi(v, q_L^W) - \pi_(v, q_H^W) \le 0$ if and only if $T^W(\alpha)$. □.

*Proof of Lemma C.4.* Taking the derivative of $\hat{\pi}_H^B$ with respect to $q$,

$$\frac{d\hat{\pi}_H^B}{dq} = \frac{\partial \pi_H^B}{\partial q} + \frac{\partial \pi_H^B}{\partial p}\frac{\partial p}{\partial q} = (p-c) - (p-b)F(q) - (s-b)f(q)\left[q - \int_{d_l}^{q}(q-x)dF(x)\right]. \tag{32}$$

As $p = v - (s-b)F(q)$. Therefore, $\frac{\partial p}{\partial q} = -(s-b)f(q)$, and hence,

$$\frac{d\hat{\pi}_H^B}{dq} = \bar{F}(q)\left[(p-b) - (s-b)h(q)\int_{d_l}^{q}\bar{F}(x)dx\right] - (c-b). \tag{33}$$

According to IFR, $h(q)$ increases in $q$ and, hence, $\frac{d\hat{\pi}_H^B}{dq}$ decreases in $q$ when $\frac{d\hat{\pi}_H^B}{dq} \ge 0$. Therefore, $\hat{\pi}_H^B$ is pseudo-concave. □

*Proof of Lemma C.5.* Similarly to the proof of Lemma C.3, we can show that for $(p_1, q)$ and $(p_2, q)$ that are both in $\Omega^B$ and with $p_1 < p_2$, the profit of $(p_2, q)$ is higher than that of $(p_1, q)$. Furthermore, as $\Omega^B$ is a closed set, for any $(p,q) \in \Omega^B$ but $p < v$ and $q < Q_b(p)$, $\exists\ \epsilon > 0$ such that $(p+\epsilon, q) \in \Omega^B$. Therefore, $(p,q) \in \Omega^B$ can be optimal only if $p = v$ or $q = Q_b(p)$.

Furthermore, for $(p,q)$ with $p < v$ and $q = Q_b(p)$, assume $\frac{dQ_b(p)}{dp} \ge 0$. $\exists\ \epsilon > 0$ such that $(p+\epsilon, q) \in \Omega^B$ and the corresponding profit is higher than $(p,q)$. Therefore, $(p,q)$ is optimal only if $\frac{dQ_b(p)}{dp} < 0$. □

*Proof of Lemma C.6.* According to Lemma C.5, for $\tau \ge T^D(\alpha)$, $(p^{B,*}, q^{B,*})$ need to satisfy $q^{B,*} = Q_b(p^{B,*})$. Depending on the relative magnitude of $q^{B,*}$, $d_\tau^B$, and $d_\tau^W$, we can discuss three further scenarios.

1. If $q^{B,*} \ge d_\tau^W$, the relative profit function is $\pi_L^B()$ and $q^{B,*} = \frac{(1-\alpha)p^{B,*}Q_s(p^{B,*}) - \tau}{c}$. Let the optimal solution be $(p_{LL}^B, q_{LL}^B)$ and the corresponding profit under this solution be $\Pi_{LL}^B(\tau,\alpha)$ for $(\tau,\alpha)$. It is clear that $\frac{d\Pi_{LL}^B}{d\tau} < 0$ and $\frac{d\Pi_{LL}^B}{d\alpha} < 0$.

2. If $d_\tau^W > q^{B,*} > d_\tau^B$, the relative profit function is $\pi_L^B()$ and $q^{B,*} = Q_s(p^{B,*})$. Let the optimal solution be $(p_{LH}^B, q_{LH}^B)$ and the corresponding profit under this solution be $\Pi_{LH}^B(\tau,\alpha)$ for $(\tau,\alpha)$. It is clear that $\frac{d\Pi_{LH}^B}{d\tau} < 0$ and $\frac{d\Pi_{LH}^B}{d\alpha} = 0$.

3. If $d_\tau^B \ge q^{B,*}$: In this scenario, the relative profit function is $\pi_H^B()$, and $q_{B,*} = Q_s(p^{B,*})$. According to Lemma C.4, the global optima is determined by $\frac{d\hat{\pi}_H^B}{dq} = 0$, i.e. $q^{B,*} = q_H^B$, and $p^{B,*} = v - (s-b)F(q_H^B)$, and the corresponding profit under this solution is $\Pi_H^B(\tau,\alpha)$ for $(\tau,\alpha)$. It is clear that $\frac{d\Pi_H^B}{d\tau} = \frac{d\Pi_H^B}{d\alpha} = 0$.

Comparing the optimal profits under the three scenarios, as $\Pi_H^B(\tau,\alpha)$ is independent of $\tau$ and $\Pi_{LL}^B(\tau,\alpha)$ decreases in $\tau$, there exist thresholds $T^{B,L}(\alpha) > T^D(\alpha)$ such that $\Pi_H^B(\tau,\alpha) \geq \Pi_{LL}^B(\tau,\alpha)$ if and only if $\tau \geq T^{B,L}(\alpha)$. Similarly, there exist thresholds $T^{B,H}(\alpha)$ such that $\Pi_H^B(\tau,\alpha) \geq \Pi_{LH}^B(\tau,\alpha)$ if and only if $\tau \geq T^{B,H}(\alpha)$.

To compare $\Pi_{LL}^B(\tau,\alpha)$ and $\Pi_{LH}^B(\tau,\alpha)$, we consider $q$ as a function of $p$. According to the envelope theorem, $\frac{d\Pi_{LL}^B - d\Pi_{LH}^B}{d\tau} = \frac{\partial \pi_L^B}{\partial q}\left(-\frac{1}{c}\right) < 0$. In addition, note that $\Pi_{LH}^B < \max[v-c-(s-b)F(q)]\int_{d_l}^{q}\bar{F}(x)dx - (c-s)q < (v-c)\int_{d_l}^{q^{NV}}\bar{F}(x)dx - (c-s)q^{NV}$. Therefore, there exists a threshold $T^{B,LL}(\alpha)$ such that $\Pi_{LL}^B \leq \Pi_{LH}^B$ if and only if $\tau \geq T^{B,LL}(\alpha)$.

Finally, given the above result, we can construct $T^{B,1}(\alpha) = \min\left(T^{B,LL}(\alpha), T^{B,L}(\alpha)\right)$ and $T^{B,2}(\alpha) = \max\left(T^{B,L}(\alpha), T^{B,H}(\alpha)\right)$ to obtain the results as stated. □

*Proof of Lemma C.7.* First, note that the function $(s-b)F(x) + tF\left(\frac{\tau+cx}{p}\right)$ increases in $x$. When $Q_s^{dd} \geq \frac{\tau}{p-c}$, we have $Q_s^{dd} \geq \frac{\tau + cQ_s^{dd}}{p}$; therefore,

$$(s-b)F\left(Q_s^{dd}\right) + tF\left(\frac{\tau + cQ_s^{dd}}{p}\right) \leq (t+s-b)F\left(Q_s^{dd}\right) = v-p+t. \tag{34}$$

Therefore, $Q_s^{dd,a} \geq Q_s^{dd}$. All of the above steps can be reversed, and hence, the first and second conditions are equivalent. Similarly, we can show that the first and third statements are equivalent. □

*Proof of Lemma C.8.* By the definition of $Q_s^{dd}$, we have:

$$(s-b)F(Q_s^{dd}) + tF[(1-\alpha)Q_s^{dd}] < (s-b+t)F(Q_s^{dd}) = v-p+t. \tag{35}$$

Therefore, $Q_s^{dd} \leq \frac{cQ_s^{dd,b}+\tau}{(1-\alpha)p}$. That is, $Q_s^{dd,b} \geq \frac{(1-\alpha)pQ_s^{dd}-\tau}{c}$. Similarly, we can show that $Q_s^{dd,b} \leq \frac{pQ_s^{dd}-\tau}{c}$. □

*Proof of Lemma C.9.* This follows directly from the definition. For example, to show that $\tau \leq [(1-\alpha)p - c]Q_s^{dd,a}$ leads to $Q_s^{dd,b} \geq Q_s^{dd,a}$, note that as $Q_s^{dd,a} > \frac{\tau}{(1-\alpha)p-c}$, we have $Q_s^{dd,a} > \frac{cQ_s^{dd,a}+\tau}{(1-\alpha)p}$. Therefore,

$$(s-b)F\left[\frac{cQ_s^{dd,a}+\tau}{(1-\alpha)p}\right] + tF\left(\frac{cQ_s^{dd,a}+\tau}{p}\right) < (s-b)F\left(Q_s^{dd,a}\right) + tF\left(\frac{cQ_s^{dd,a}+\tau}{p}\right) = v-p+t. \tag{36}$$

Therefore, $Q_s^{dd,b} \geq Q_s^{dd,a}$, as desired. □

*Proof of Lemma C.10.* In this case, $q < d_\tau^B$, customers' expected surplus in the buy-equilibrium is $v-p$. In the wait-equilibrium, it is $(s-b)F\left(\frac{cq+\tau}{(1-\alpha)p}\right)$. Therefore, the buy-equilibrium is more appealing if and only if $v-p \geq (s-b)F(q)$ or, equivalently, $q \leq Q_s$. This is exactly the condition under which the wait-equilibrium exists. Therefore, customers prefer the wait-equilibrium.

When $q \geq d_\tau^B$, customers' expected surplus in the buy-equilibrium is $v-p+t\left[1-F\left(\frac{cq+\tau}{p}\right)\right]$. In the wait-equilibrium, it is $(s-b)F(q)$. Therefore, the buy-equilibrium is more appealing if and only if

$$v-p+t\left[1-F\left(\frac{cq+\tau}{p}\right)\right] \geq (s-b)F(q) \tag{37}$$

or, equivalently, $q \leq Q_s^{dd,a}$.

Finally, when $q \geq d_\tau^B$, customers' expected surplus in the buy-equilibrium is $v-p+t\left[1-F\left(\frac{cq+\tau}{p}\right)\right]$. In the wait-equilibrium, it is $(s-b)F\left(\frac{cq+\tau}{(1-\alpha)p}\right)$. Therefore, the buy-equilibrium is more appealing to customers if and only if

$$v-p+t\left[1-F\left(\frac{cq+\tau}{p}\right)\right] \geq (s-b)F\left(\frac{cq+\tau}{(1-\alpha)p}\right) \tag{38}$$

or, equivalently, $q \leq Q_s^{dd,b}$. □

*Proof of Lemma C.11.* In preparation, we rearrange the existence condition for the buy-equilibrium in Statement 1 of Proposition 4 by considering the following three scenarios.

1. When $\tau \le (p-c)Q_s$, $\left(\frac{\tau}{p-c}, \frac{pQ_s^{dd}-\tau}{c}\right] \ne \emptyset$, and $\min\left(Q_s, \frac{\tau}{p-c}\right) = \frac{\tau}{p-c}$. Therefore, in this region, the buy-equilibrium exists if and only if $q \le \frac{pQ_s^{dd}-\tau}{c}$.

2. When $\tau \in [(p-c)Q_s, (p-c)Q_s^{dd}]$, $\left(\frac{\tau}{p-c}, \frac{pQ_s^{dd}-\tau}{c}\right] \ne \emptyset$. And $\min\left(Q_s, \frac{\tau}{p-c}\right) = Q_s$. Therefore, in this region, the buy-equilibrium exists if and only if $q \le Q_s$ or $q \in \left(\frac{\tau}{p-c}, \frac{pQ_s^{dd}-\tau}{c}\right]$.

3. When $\tau > (p-c)Q_s^{dd}$, $\left(\frac{\tau}{p-c}, \frac{pQ_s^{dd}-\tau}{c}\right] = \emptyset$. Note that as $Q_s^{dd} \ge Q_s$, $\min\left(Q_s, \frac{\tau}{p-c}\right) = Q_s$. Therefore, in this region, the buy-equilibrium exists if and only if $q \le Q_s$.

Symmetrically, we rearrange the existence condition for the wait-equilibrium in Statement 2 of Proposition 4 by considering the following three scenarios.

1. When $\tau > [(1-\alpha)p - c]Q_s^{dd}$, $\left(Q_s, \frac{\tau}{(1-\alpha)p-c}\right) \ne \emptyset$, and $\frac{\tau}{(1-\alpha)p-c} > \frac{(1-\alpha)pQ_s^{dd}-\tau}{c}$. Therefore, the wait-equilibrium exists if and only if $q > Q_s$.

2. When $\tau \in [[(1-\alpha)p-c]Q_s, [(1-\alpha)p-c]Q_s^{dd}]$, $\left(Q_s, \frac{\tau}{(1-\alpha)p-c}\right) \ne \emptyset$, and $\frac{\tau}{(1-\alpha)p-c} \le \frac{(1-\alpha)pQ_s^{dd}-\tau}{c}$. Therefore, the wait-equilibrium exists if and only if $q > \frac{(1-\alpha)pQ_s^{dd}-\tau}{c}$ or $q \in \left(Q_s, \frac{\tau}{(1-\alpha)p-c}\right)$.

3. When $\tau < [(1-\alpha)p-c]Q_s$, $\left(Q_s, \frac{\tau}{(1-\alpha)p-c}\right) = \emptyset$, and as $Q_s \le Q_s^{dd}$, $\frac{\tau}{(1-\alpha)p-c} \le \frac{(1-\alpha)pQ_s^{dd}-\tau}{c}$. Therefore, the wait-equilibrium exists if and only if $q > \frac{(1-\alpha)pQ_s^{dd}-\tau}{c}$.

According to the above conditions, note that when $\tau > (p-c)Q_s^{dd}$, the buy-equilibrium exists if and only if $q \le Q_s$ and the wait-equilibrium exists if any only if $q > Q_s$. This is consistent with the no-deferred-discount case, in which multiple equilibria do not exist when $\tau > (p-c)Q_s$. Therefore, we only need to consider the region with $\tau \le (p-c)Q_s^{dd}$. We further divide this region into two cases.

**Case I:** $\tau \le [(1-\alpha)p-c]Q_s^{dd,a}$**.** In this region, we have $\tau \le (p-c)Q_s^{dd,a}$ and, according to Lemma C.7, $\tau \le (p-c)Q_s^{dd}$. Second, according to Lemma C.9, as $\tau \le [(1-\alpha)p-c]Q_s^{dd,a}$, $Q_s^{dd,a} \le Q_s^{dd,b}$. With these properties, we further divide $\tau \le [(1-\alpha)p-c]Q_s^{dd,a}$ into the following regions depending on the conditions for the buy- and wait-equilibria to exist as specified above.

1. $\tau \le [(1-\alpha)p-c]Q_s$, the buy- and wait-equilibria both exist if and only if $q \in \left(\frac{(1-\alpha)pQ_s^{dd}-\tau}{c}, \frac{pQ_s^{dd}-\tau}{c}\right]$, which is not empty for $\alpha > 0$. For customers to choose the buy-equilibrium, note that for $q > \frac{(1-\alpha)pQ_s^{dd}-\tau}{c}$, we have $q > Q_s^{dd}$, and hence, $[(1-\alpha)p-c]q > \tau$. Therefore, $q \ge \frac{cq+\tau}{(1-\alpha)p} = d_\tau^W > d_\tau^B$. Hence, the buy-equilibrium is more appealing to customers if and only if $q \le Q_s^{dd,b}$, which falls in this region.

2. $\tau \in ([(1-\alpha)p-c]Q_s, [(1-\alpha)p-c]Q_s^{dd}]$: In this region, the wait-equilibrium exists if and only if $q > \frac{(1-\alpha)pq_s^{dd}-\tau}{c}$ or $q \in \left(Q_s, \frac{\tau}{(1-\alpha)p-c}\right)$. However, the region for the buy-equilibrium to exist depends on the relative magnitude of $(p-c)Q_s$ and $[(1-\alpha)p-c]Q_s^{dd}$. We consider three further sub-scenarios.

   (a) $(p-c)Q_s < [(1-\alpha)p-c]Q_s^{dd}$ and $\tau \in ([(1-\alpha)p-c]Q_s, (p-c)Q_s)$: Both equilibria exist when $q \in \left(Q_s, \frac{\tau}{(1-\alpha)p-c}\right]$ or $q \in \left[\frac{(1-\alpha)pQ_s^{dd}-\tau}{c}, \frac{pQ_s^{dd}-\tau}{c}\right]$. In the first sub-region, $q \in [d_\tau^B, d_\tau^W]$. Therefore, customers prefer the buy-equilibrium if and only if $q \le Q_s^{dd,a}$. Note that as $Q_s^{dd,a} \ge \frac{\tau}{(1-\alpha)p-c}$, customers always prefer the buy-equilibrium in this sub-region. In the second sub-region, similarly to the case with $\tau < [(1-\alpha)p-c]Q_s$, customers prefer the buy-equilibrium if and only if $q \le Q_s^{dd,b}$. Combining the two sub-regions, customers prefer the buy-equilibrium if and only if $q \le Q_s^{dd,b}$.

(b) $(p-c)Q_s < [(1-\alpha)p-c]Q_s^{dd}$ and $\tau \in ((p-c)Q_s, [(1-\alpha)p-c]Q_s^{dd})$: Both equilibria exist when $q \in \left(\frac{\tau}{p-c}, \frac{\tau}{(1-\alpha)p-c}\right)$ or $q \in \left[\frac{(1-\alpha)pQ_s^{dd}-\tau}{c}, \frac{pQ_s^{dd}-\tau}{c}\right]$. In the first sub-region, $q \in [d_\tau^B, d_\tau^W]$. Therefore, customers prefer the buy-equilibrium if and only if $q \le Q_s^{dd,a}$. Similarly to the previous case, we again have $Q_s^{dd,a} > \frac{\tau}{(1-\alpha)p-c}$, and hence, customers always prefer the buy-equilibrium in the first sub-region. In the second sub-region, similarly to the case with $\tau \le [(1-\alpha)p-c]Q_s$, customers prefer the buy-equilibrium if and only if $q \le Q_s^{dd,b}$. Combining the two sub-regions, customers prefer the buy-equilibrium if and only if $q \le Q_s^{dd,b}$.

(c) $(p-c)Q_s \ge [(1-\alpha)p-c]Q_s^{dd}$ and $\tau \in ([(1-\alpha)p-c]Q_s, [(1-\alpha)p-c]Q_s^{dd})$: In this region, both equilibria co-exist if and only if $q \in \left(Q_s, \frac{\tau}{(1-\alpha)p-c}\right)$ or $q \in \left(\frac{(1-\alpha)pQ_s^{dd}-\tau}{c}, \frac{pQ_s^{dd}-\tau}{c}\right]$. In the first sub-region, $q < d_\tau^W$. As $q > Q_s > \frac{\tau}{p-c}$, we have $q > d_\tau^B$. Therefore, customers prefer the buy-equilibrium if and only if $q \le Q_s^{dd,a}$. Therefore, in the first sub-region, customers always prefer the buy-equilibrium. In the second sub-region, that is, $q \in \left(\frac{(1-\alpha)pQ_s^{dd}-\tau}{c}, \frac{pQ_s^{dd}-\tau}{c}\right]$, similarly to the first case, we have $q > d_\tau^W$. Therefore, customers prefer the buy-equilibrium if and only if $q \le Q_s^{dd,b}$.

3. $\tau \in ([(1-\alpha)p-c]Q_s^{dd}, [(1-\alpha)p-c]Q_s^{dd,a}]$: In this region, the wait-equilibrium exists if and only if $q > Q_s$. However, the region for the buy-equilibrium to exist depends on the relative magnitude of $(p-c)Q_s$ and $[(1-\alpha)p-c]Q_s^{dd}$ and $[(1-\alpha)p-c]Q_s^{dd,a}$. Therefore, we consider four further sub-scenarios.

(a) $(p-c)Q_s < [(1-\alpha)p-c]Q_s^{dd}$ and $\tau \in ([(1-\alpha)p-c]Q_s^{dd}, [(1-\alpha)p-c]Q_s^{dd,a})$: As $Q_s < \frac{\tau}{p-c}$, both equilibria exist if and only if $q \in \left(\frac{\tau}{p-c}, \frac{pQ_s^{dd}-\tau}{c}\right)$. In this region, $q > d_\tau^B$. However, as $\frac{\tau}{(1-\alpha)p-c} < \frac{pQ_s^{dd}-\tau}{c}$, the relative magnitude of $q$ and $d_\tau^W$ is undetermined in this region. Therefore, we further divide this region into two sub-regions: $q \in \left(\frac{\tau}{p-c}, \frac{\tau}{(1-\alpha)p-c}\right)$ and $q \in \left[\frac{\tau}{(1-\alpha)p-c}, \frac{pQ_s^{dd}-\tau}{c}\right)$. In the first sub-region, $q \in [d_\tau^B, d_\tau^W]$, and hence, customers prefer the buy-equilibrium if and only if $q \le Q_s^{dd,a}$. As $Q_s^{dd,a} \ge \frac{\tau}{(1-\alpha)p-c}$, customers always prefer the buy-equilibrium in this sub-region. In the second sub-region, $q > d_\tau^W$, and hence, customers prefer the buy-equilibrium if and only if $q \le Q_s^{dd,b}$. Note that as $Q_s^{dd,b} > \frac{\tau}{(1-\alpha)p-c}$, and $Q_s^{dd,b} < \frac{pQ_s^{dd}-\tau}{c}$, customers prefer the buy-equilibrium if and only if $q \le Q_s^{dd,b}$ in the second sub-region.

(b) $(p-c)Q_s \ge [(1-\alpha)p-c]Q_s^{dd,a}$ and $\tau \in ([(1-\alpha)p-c]Q_s^{dd}, [(1-\alpha)p-c]Q_s^{dd,a})$: Both equilibria exist if and only if $q \in \left(Q_s, \frac{pQ_s^{dd}-\tau}{c}\right]$. As $\frac{pQ_s^{dd}-\tau}{c} > Q_s^{dd} \ge Q_s$, this interval is not empty. As $q > Q_s > \frac{\tau}{p-c}$, $q > d_\tau^B$. However, note that $\left(p - \frac{c}{1-\alpha}\right)Q_s^{dd} > (p-c)Q_s^{dd} > \tau$; therefore, $\frac{\tau}{(1-\alpha)p-c} < \frac{pQ_s^{dd}-\tau}{c}$, and hence, the relative magnitude of $q$ and $d_\tau^W$ is undetermined in this region. Therefore, we further divide this region into two sub-regions: $q \in \left(Q_s, \frac{\tau}{(1-\alpha)p-c}\right)$ and $q \in \left[\frac{\tau}{(1-\alpha)p-c}, \frac{pQ_s^{dd}-\tau}{c}\right]$. In the first sub-region, $q < d_\tau^W$, and hence, customers prefer the buy-equilibrium when $q \le Q_s^{dd,a}$, which is greater than $\frac{\tau}{(1-\alpha)p-c}$. In the second sub-region, $q \ge d_\tau^W$, and hence, customers prefer the buy-equilibrium when $q \le Q_s^{dd,b}$, which falls between $\frac{\tau}{(1-\alpha)p-c}$ and $\frac{pQ_s^{dd}-\tau}{c}$. Therefore, in this region, customers prefer the buy-equilibrium if and only if $q \le Q_s^{dd,b}$.

(c) $(p-c)Q_s \in [[(1-\alpha)p-c]Q_s^{dd}, [(1-\alpha)p-c]Q_s^{dd,a})$, and $\tau \in ([(1-\alpha)p-c]Q_s^{dd}, (p-c)Q_s)$: This scenario has exactly the same result as in the scenario with $(p-c)Q_s \ge [(1-\alpha)p-c]Q_s^{dd,a}$ and $\tau \in ([(1-\alpha)p-c]Q_s^{dd}, [(1-\alpha)p-c]Q_s^{dd,a})$, and hence, customers prefer the buy-equilibrium if and only if $q \le Q_s^{dd,b}$.

(d) $(p-c)Q_s \in [[(1-\alpha)p-c]Q_s^{dd}, [(1-\alpha)p-c]Q_s^{dd,a})$, and $\tau \in ((p-c)Q_s, [(1-\alpha)p-c]Q_s^{dd,a})$: As $Q_s < \frac{\tau}{p-c}$, both equilibria exist if and only if $q \in \left(\frac{\tau}{p-c}, \frac{pQ_s^{dd}-\tau}{c}\right]$, which obviously is not empty. In this interval, similarly to the previous case, $q > d_\tau^B$. Again, we divide the region into two sub-regions: $q \in \left(\frac{\tau}{p-c}, \frac{\tau}{(1-\alpha)p-c}\right)$

and $q \in \left[\frac{\tau}{(1-\alpha)p-c}, \frac{pQ_s^{dd}-\tau}{c}\right]$. As in the previous case, customers always prefer the buy-equilibrium in the first sub-region. In the second sub-region, customers prefer the buy-equilibrium if and only if $q \le Q_s^{dd,b}$.

Summarizing the above scenarios, we can conclude that when $\tau \le [(1-\alpha)p-c]Q_s^{dd,a}$, the buy-equilibrium is more appealing to customers if and only if $q \le Q_s^{dd,b}$.

**Case II:** $\tau \in ([(1-\alpha)p-c]Q_s^{dd,a}, (p-c)Q_s^{dd}]$**.** In this region, according to Lemma C.9, $\frac{\tau}{(1-\alpha)p-c} > Q_s^{dd,a} > Q_s^{dd,b}$. Furthermore, in this region, the wait-equilibrium exists if and only if $q > Q_s$. However, similarly to the previous case, the existence region for the buy-equilibrium depends on the relative magnitude of $(p-c)Q_s$ and $[(1-\alpha)p-c]Q_s^{dd,a}$. This requires us to consider three sub-scenarios.

1. $(p-c)Q_s \le [(1-\alpha)p-c]Q_s^{dd,a}$ and $\tau \in ([(1-\alpha)p-c]Q_s^{dd,a}, (p-c)Q_s^{dd})$: Both equilibria exist if and only if $q \in \left(\frac{\tau}{p-c}, \frac{pQ_s^{dd}-\tau}{c}\right)$. The only difference between this scenario and that with $(p-c)Q_s \ge [(1-\alpha)p-c]Q_s^{dd,a}$ and $\tau \in ([(1-\alpha)p-c]Q_s^{dd}, [(1-\alpha)p-c]Q_s^{dd,a})$ is that $Q_s^{dd,b} < Q_s^{dd,a} < \frac{\tau}{(1-\alpha)p-c}$. Therefore, customers prefer the buy-equilibrium if and only if $q \le Q_s^{dd,a}$.

2. $(p-c)Q_s > [(1-\alpha)p-c]Q_s^{dd,a}$ and $\tau \in ([(1-\alpha)p-c]Q_s^{dd,a}, (p-c)Q_s)$: Both equilibria exist if and only if $q \in \left(Q_s, \frac{pQ_s^{dd}-\tau}{c}\right]$. Similarly to the previous case, as $Q_s^{dd,b} < Q_s^{dd,a} < \frac{\tau}{(1-\alpha)p-c}$, customers prefer the buy-equilibrium if and only if $q \le Q_s^{dd,a}$.

3. $(p-c)Q_s > [(1-\alpha)p-c]Q_s^{dd,a}$ and $\tau \in ([(p-c)Q_s, (p-c)Q_s^{dd})$: Again, the only difference between this scenario and that with $(p-c)Q_s \in [[(1-\alpha)p-c]Q_s^{dd}, [(1-\alpha)p-c]Q_s^{dd,a})$, and $\tau \in ((p-c)Q_s, [(1-\alpha)p-c]Q_s^{dd,a})$ is that $Q_s^{dd,b} < Q_s^{dd,a} < \frac{\tau}{(1-\alpha)p-c}$. Therefore, customers prefer the buy-equilibrium if and only if $q \le Q_s^{dd,a}$.

Therefore, we can conclude that when $\tau \in ([(1-\alpha)p-c]Q_s^{dd,a}, (p-c)Q_s^{dd}]$, the buy-equilibrium is more appealing to customers if and only if $q \le Q_s^{dd,a}$. $\square$

## Appendix E: Details on numerical results

In the paper, we have conducted extensive numerical experiments to verify that the numerical results presented in the main body of paper are robust under different parameter specifications. Table 3 summarizes the parameters we have used to conduct our numerical results.

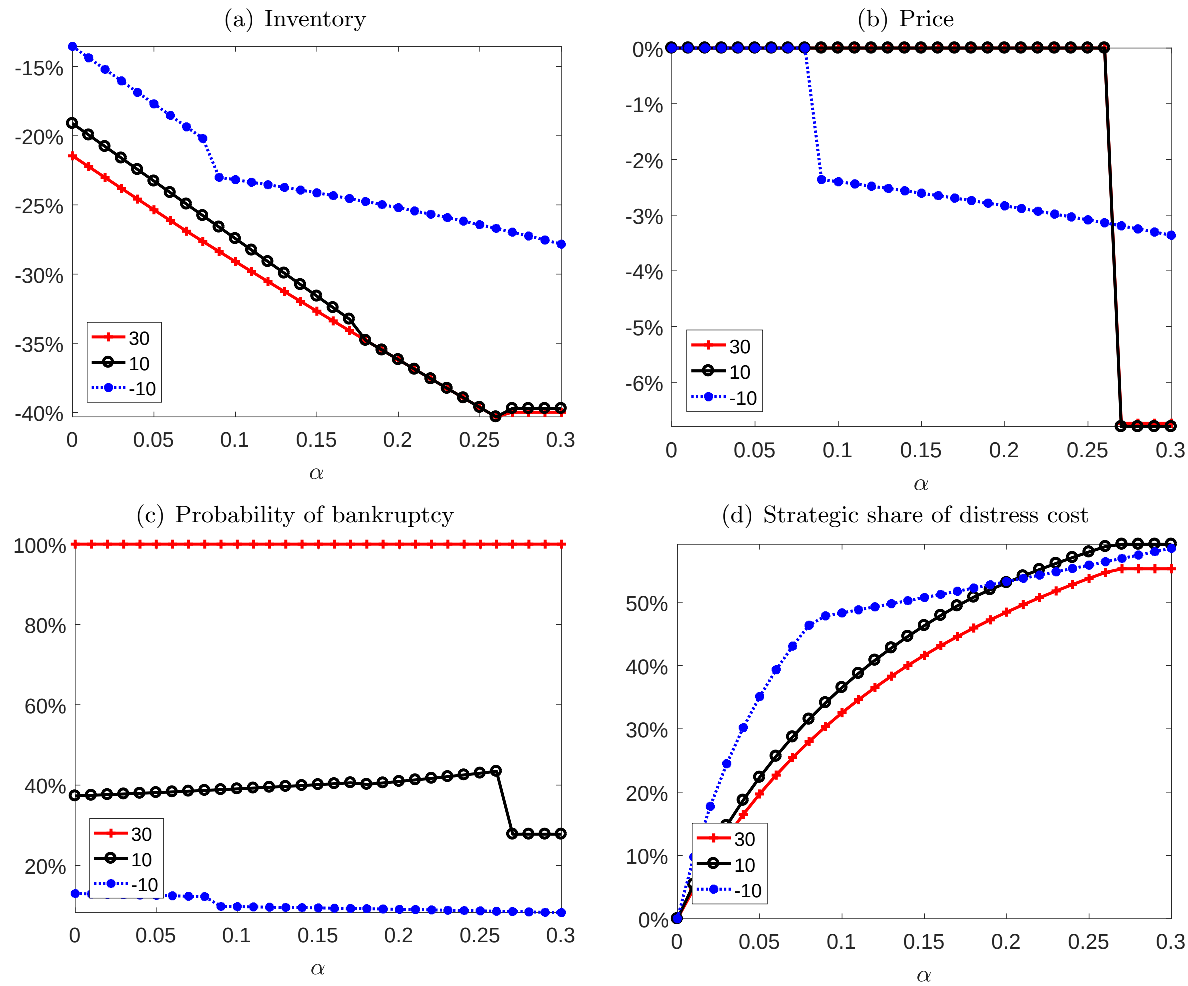
**Table 3 Parameters used in numerical experiments**

| Parameters | Values Used |
|---|---|
| Demand Distribution $D$ | Uniform[0, 300], Uniform[100,200], Uniform[80, 220], Uniform[20, 280] |
| | Triangular[0, 100, 50] (presented in the paper), Triangular[0, 100, 25], Triangular[0, 100,75], Triangular[0, 300, 50], Triangular[20, 280, 50] |
| | Truncated normal with $\mu = 150$, $\sigma = 50$, and truncated at 0 and 300 |
| | Binary with different parameters (both analytical and numerical results obtained) |
| $v$ | 1 |
| $c$ | 0.55, 0.6 (presented in the paper), 0.65, 0.7, 0.75 |
| $s$ | 0.4, 0.45, 0.5 (presented in the paper) |
| $b$ | 0.2, 0.25, 0.3 (presented in the paper), 0.35, 0.4 |
| $\alpha$ | [0, 0.5] with an increment of 0.01 |
| $\tau$ | [-60, 60] with an increment of 1 |

## Appendix F: The impact of strategic fraction ($\alpha$): An alternative illustration

In this appendix, we present the impact of strategic fraction $\alpha$ on the firm's operational and financial metrics (currently illustrated in Figures 5 in the main body of the paper) in an alternative form, as in Figures 8.

**Figure 8** **The impact of fraction of strategic customers ($\alpha$) and level of financial distress ($\tau$) on operational decisions and performance**




Notes. All $D \sim Triangular(0, 100, 50)$, $v = 1$, $c = 0.6$, $s = 0.5$, $b = 0.3$. Different lines represent different levels of financial distress ($\tau$), as marked in the legend. Figure 8(a) (8(b)) represents the inventory (price) change in percentage relative to the the newsvendor benchmark $(v, q^{NV})$. Figure 8(d) represents the strategic share of distress cost, defined as the proportion of total distress cost caused by strategic consumers, i.e. $\frac{\Pi(\tau,\alpha)-\Pi(\tau,0)}{\Pi(\tau,\alpha)-\Pi(-\infty,0)}$, where $\Pi(\tau, \alpha)$ is the firm's *optimal* profit under $(\tau, \alpha)$. In Figure 8(d), the strategic share of distress cost is not defined when $\tau$ is low as the total distress cost is zero.

As shown in Figure 8(a), regardless of $\tau$, the firm should always lower its inventory level when facing more strategic consumers, while the magnitude of the inventory reduction increases as the firm becomes more financially distressed.

The trend on pricing, as presented in Figure 8(b), however, is less straightforward. Indeed, we find that when the firm's financial distress is in the medium level (as represented by $\tau = 10$ in the figure), the firm starts to lower price even when there only exists a small fraction of strategic customers. This mirrors the boundary between Region $BL$ and $NV$ in Figure 4. However, as the firm becomes more distressed, it only offers a price discount when the strategic fraction is high, yet when the price discount is very deep. This echoes the bifurcation between $ISC$ and $BH$, and also the findings Figure 5(b).

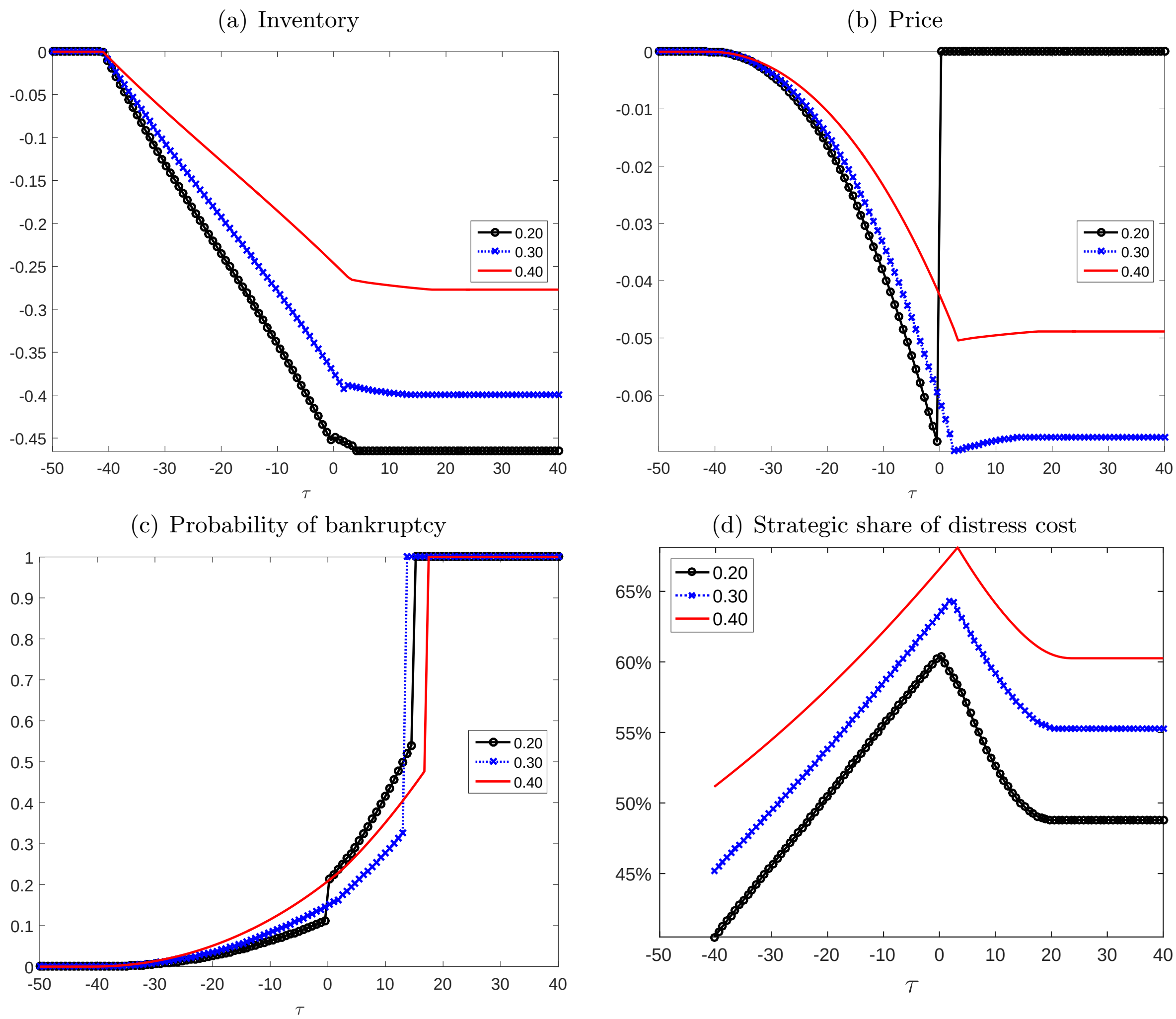
Regarding bankruptcy probability, as shown in Figure 8(c), when the firm's probability of bankruptcy may drop (as shown in the two lines with $\tau = -10$ and 10) as $\alpha$ increases. This corresponds to the changes in the firm's pricing strategy. Finally, Figure 8(d) has confirmed that as $\alpha$ increases, the part of cost of financial distress that is due to strategic customer behavior becomes more and more important.

On the impact of $\alpha$ on the magnitude and effectiveness of deferred discount, by rotating Figure 7 in a similar way, we confirm the insight we highlight – deferred discount is most valuable to the firm when the firm's financial distress is at a medium level, and the fraction of strategic consumers is high.

## Appendix G: Comparative statics on $c$, $s$, and $b$

To better understand how different parameters influence the interaction between financial distress and strategic customer behavior, we conducted extensive numerical experiments to establish comparative statics on the deep liquidation price $b$, which captures the incentive for waiting, the "regular" salvage price $s$, and the procurement cost $c$, which captures the regular profit margin. A representative set of results are illustrated in Figures 9 – 11. In all figures, we use the base parameter $D \sim Trianglar(0,100,50)$, $\alpha = 0.3$, $v = 1$, $c = 0.6$, $s = 0.5$ and $b = 0.3$. We then vary $b$, $s$ and $c$ in each figures.

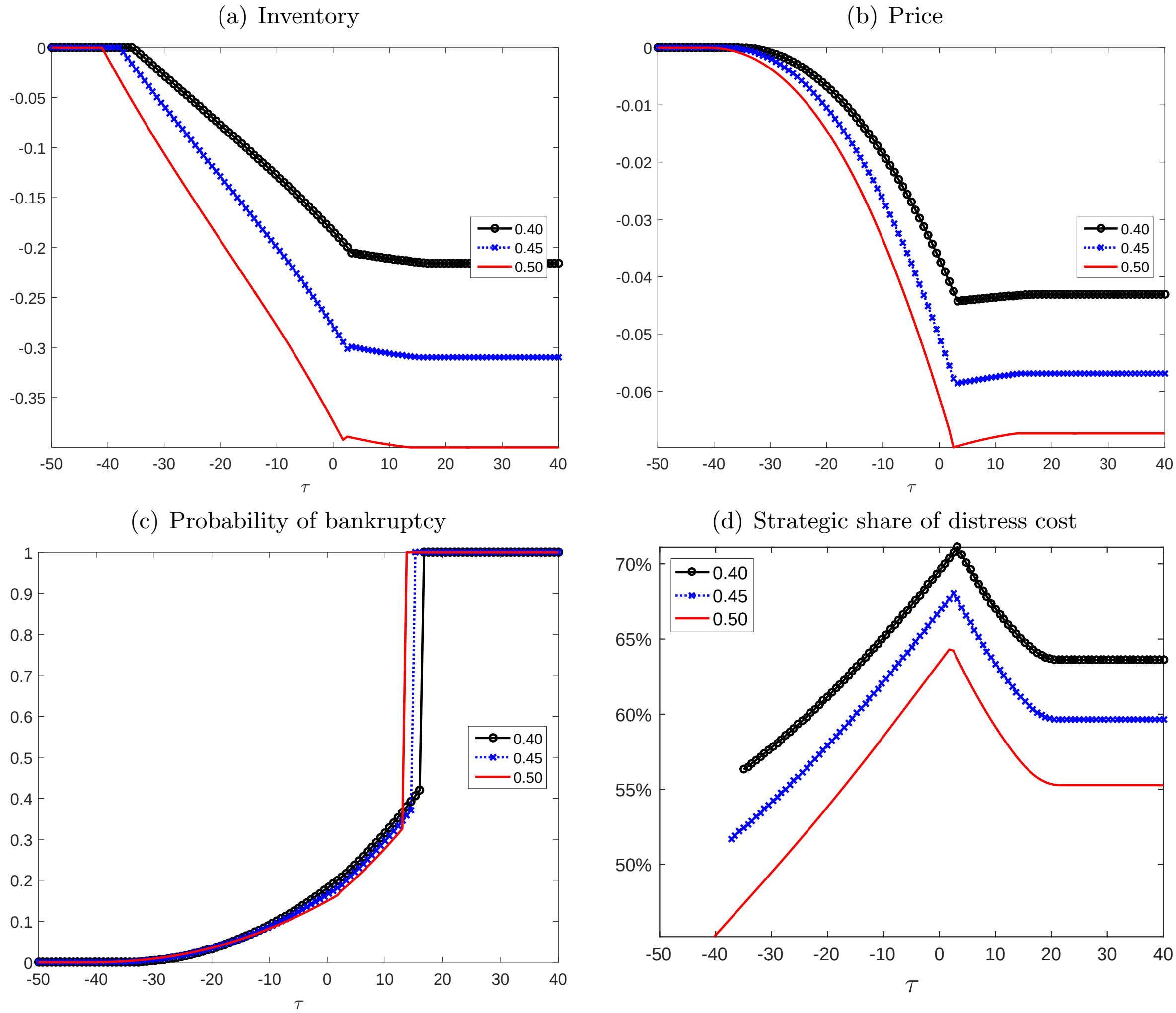
**Figure 9 The impact of $b$ on operational decisions and performance**




Notes: $D \sim Triangular(0,100,50)$, $\alpha = 0.3$, $v = 1$, $c = 0.6$, $s = 0.5$. Figures 9(a) and 9(b) capture the relative difference of inventory (and price) from the newsvendor benchmark $(v, q^{NV})$.

We start with the impact of $b$ on the firm's operational decisions and financial performance, as illustrated in Figure 9. As a parameter unique to our model, $b$ captures bargain hunters' valuation in a liquidation sale. In some sense, a high $b$ corresponds to a liquidation sale that is well-run, so that customers' deferred

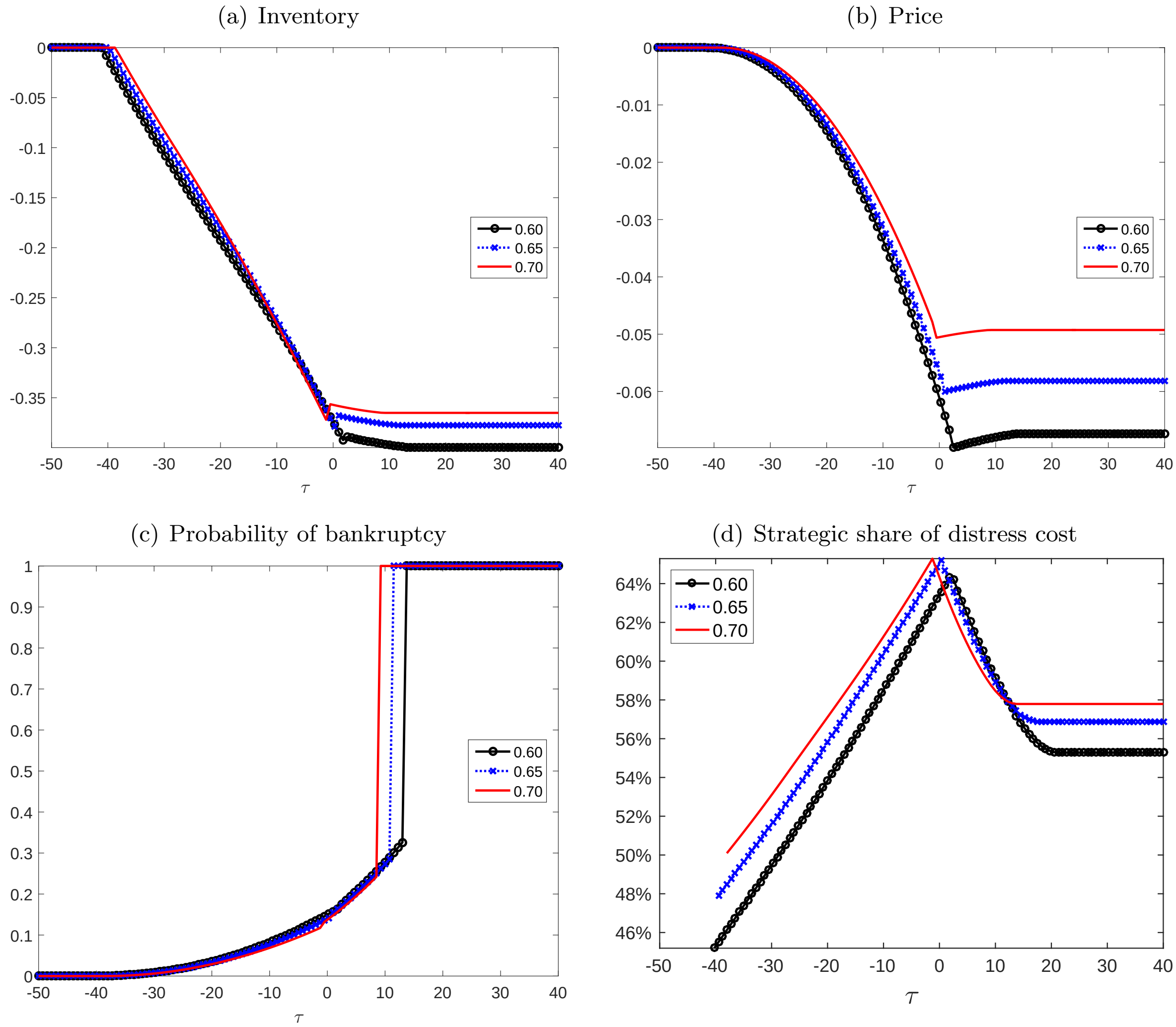
**Figure 10 The impact of $s$ on operational decisions and performance**




Notes: $D \sim Triangular(0, 100, 50)$, $\alpha = 0.3$, $v = 1$, $c = 0.6$, $b = 0.3$. Figures 10(a) and 10(b) capture the relative difference of inventory (and price) from the newsvendor benchmark $(v, q^{NV})$.

discounts in a liquidation sale does not differ too much from a regular salvage sale. As such, a higher $b$ also leads to a lower incentive for strategic customers to wait. This is exactly captured in Figure 9. As shown in Figure 9(a), a higher $b$ leads to a smaller inventory reduction. Similarly, regarding price discounts, when the level of financial distress is not too high, a higher $b$ leads to a smaller price discount. However, when $\tau$ is sufficiently high, if $b$ is sufficiently low (as captured by $b = 0.2$ in Figure 9(b)), then it is too costly for the firm to induce customers to purchase early. Instead, the optimal decision is to revert back to full price and "ignore" strategic customers. In Figure 9(c), we can see that again, for low $\tau$, because of the more aggressive inventory reduction and price discounts, a lower $b$ corresponds to a lower probability of bankruptcy. However, as $\tau$ increases, such inventory reduction and price discounts become too costly to the firm, who then actually gives up and lets the probability of bankruptcy for lower $b$ to jump up. This echoes Figure 5(c), which shows that a higher fraction of strategic customers also lead to similar behavior. Finally, on strategic share of distress cost, this is not straightforward as a decrease in $b$ increases both the "direct"

**Figure 11 The impact of $c$ on operational decisions and performance**




Notes: $D \sim Triangular(0, 100, 50)$, $\alpha = 0.3$, $v = 1$, $s = 0.5$, $b = 0.3$. Figures 11(a) and 11(b) capture the relative difference of inventory (and price) from the newsvendor benchmark $(v, q^{NV})$.

and "strategic" portion of distress cost. Indeed, as shown in Figure 9(d), as $b$ increases, the strategic share of distress cost actually increases, suggesting that the existence of strategic customers may actually be more harmful (relatively) to those firms that can run liquidation sales more efficiently.

The impact of $s$ on the firm's operational decisions and performance is illustrated in Figure 10. On inventory reduction (Figure 10(a)), first note that unlike the above case with $b$, which does not influence the newsvendor benchmark $q^{NV}$, a larger $s$ should lead to a greater $q^{NV}$. Similarly, a greater $s$ also increases the "average" salvage price the firm faces, and hence, even in the presence of financial distress, the above effect should lead to a higher inventory level. However, as the difference between $s$ and $b$ also determines the strategic customers' waiting incentive, an increase in $s$ also suggests that the firm needs to cut inventory more aggressively in order to induce strategic customers to purchase. Balancing the above two forces, Figure 10(a) shows that the inventory reduction (relative to the corresponding newsvendor quantity $q^{NV}$) increases as $s$

increases, suggesting that the strategic waiting channel dominates. For the same reason, as shown in Figure 10(b), a higher $s$ also leads to a larger price discount. The impact of $s$ on the probability of bankruptcy, as illustrated in Figure 10(c), also shows a similar pattern: A larger $s$ leads to more aggressive inventory reduction and price discounts, and hence results in a smaller probability of bankruptcy for low $\tau$, but it also leads to an earlier jump to a high probability of bankruptcy as $\tau$ increases. On strategic share of distress cost, as shown in Figure 10(d), similar to Figure Figure 9(d), financial distress caused by strategic customers are relatively more important when $s$ is small, or equivalently, when the overall distress is less severe.

Finally, Figure 11 illustrates the impact of $c$ on the firm's operational decisions and performance. For its impact on inventory, note that similar to $s$, $c$ also has a direct impact on the newsvendor benchmark $q^{NV}$. Specifically, a greater $c$ lowers $q^{NV}$. In addition, $c$ also influences the firm's financial distress through two competing forces: First, a larger $c$ suggests a lower profit margin, leading to more severe financial distress. Second, a larger $c$ results in a smaller inventory level, alleviating financial distress. The aggregated effect, however, is less clear. The above impact also interacts with the waiting incentive of strategic customers: while a low inventory level weakens customers' incentive to wait, more severe financial distress strengthens it. Combining all the above effects, Figure 11(a) shows that as $c$ increases, the (relative) inventory reduction decreases, i.e. the firm becomes less aggressive in cutting inventory. This suggests that the effect of inventory reduction associated with the increase in $c$ dominates the financial distress. For the same reason, the price discount offered by the firm also decreases in $c$. Based on the impact of $c$ on inventory and price, the firm's probability of bankruptcy, as shown in Figures 11(c) follows the same pattern as in the case with $b$ and $s$. Finally, Figure 11(d) suggests that the strategic share of distress cost is slightly higher when $c$ is lower, also consistent with the pattern with $s$ and $b$.

## Appendix H: Rational expectations equilibrium with unobservable $q$

In the main body of the paper, we assume that customers observe the inventory level $q$. This is used in Liu and van Ryzin (2008), and is examined as the quantity commitment in Su and Zhang (2008, 2009). We adopt this approach as the basic model mainly because of our focus on how strategic customers behavior can aggravate financial distress. As we already know from Su and Zhang (2008, 2009), committing on quantity improves the firm's profit. Therefore, focusing on such a scenario, we highlight that even when the firm adopts valuable operational strategies to mitigate strategic customer behavior, such behavior still can account for a significant portion of the firm's distress cost, and other mitigating mechanisms such as deferred discounts can still improve the firm's profitability.

To show the robustness of our main results, in this Appendix, we examine the case when customers observe only price $p$, but not inventory $q$. This is consistent with the rational expectations equilibria framework used in the base model in Su and Zhang (2008) and Cachon and Swinney (2009). In the following, we present the basic steps to establish the rational expectations equilibrium, and then use a set of numerical results to illustrate the robustness of the insights we highlighted in the paper.

Under this framework, after observing the firm's price $p$, strategiccustomers form a rational belief on its inventory level, which we denote as $\hat{q}$, and make their purchasing decisions based on the belief $\hat{q}$. Customers' purchasing behavior is summarized in the following result.

Lemma H.1. Under the belief that the inventory is $\hat{q}$, strategic customers purchase if and only if

$$\hat{q} \le Q_b(p) := \max\left(\frac{(1-\alpha)pQ_s(p)-\tau}{c},\, Q_s(p)\right), \tag{39}$$

where $Q_s(p) = F^{-1}\left(\frac{v-p}{s-b}\right)$.

As shown under a certain belief on the firm's inventory level $\hat{q}$, the customers' purchasing behavior is exactly the same as we identify in the main body of the paper. Therefore, the self-fulfilling property of bankruptcy, as highlighted in the paper, is also preserved.

On the firm side, there are two possibilities depending on the firm's belief of strategic customers' purchasing decisions.

1. If the firm believes that strategic customers wait, the analysis is the same as what we have in the main body of the paper, i.e. the price is $v$, and the optimal inventory $q$ is the same as in the optimal wait-region inventory. Let the corresponding firm's profit to be $\Pi_W^*$.
2. If the firm believes that strategic customers buy, under any $p$, it solves $q$ to maximize:

$$\pi(q;p) = (p-c)q - (p-s)\int_{d_l}^{q}(q-x)dF(x) - (s-b)\int_{d_l}^{\min(q,d_\tau^B)}(q-x)dF(x). \tag{40}$$

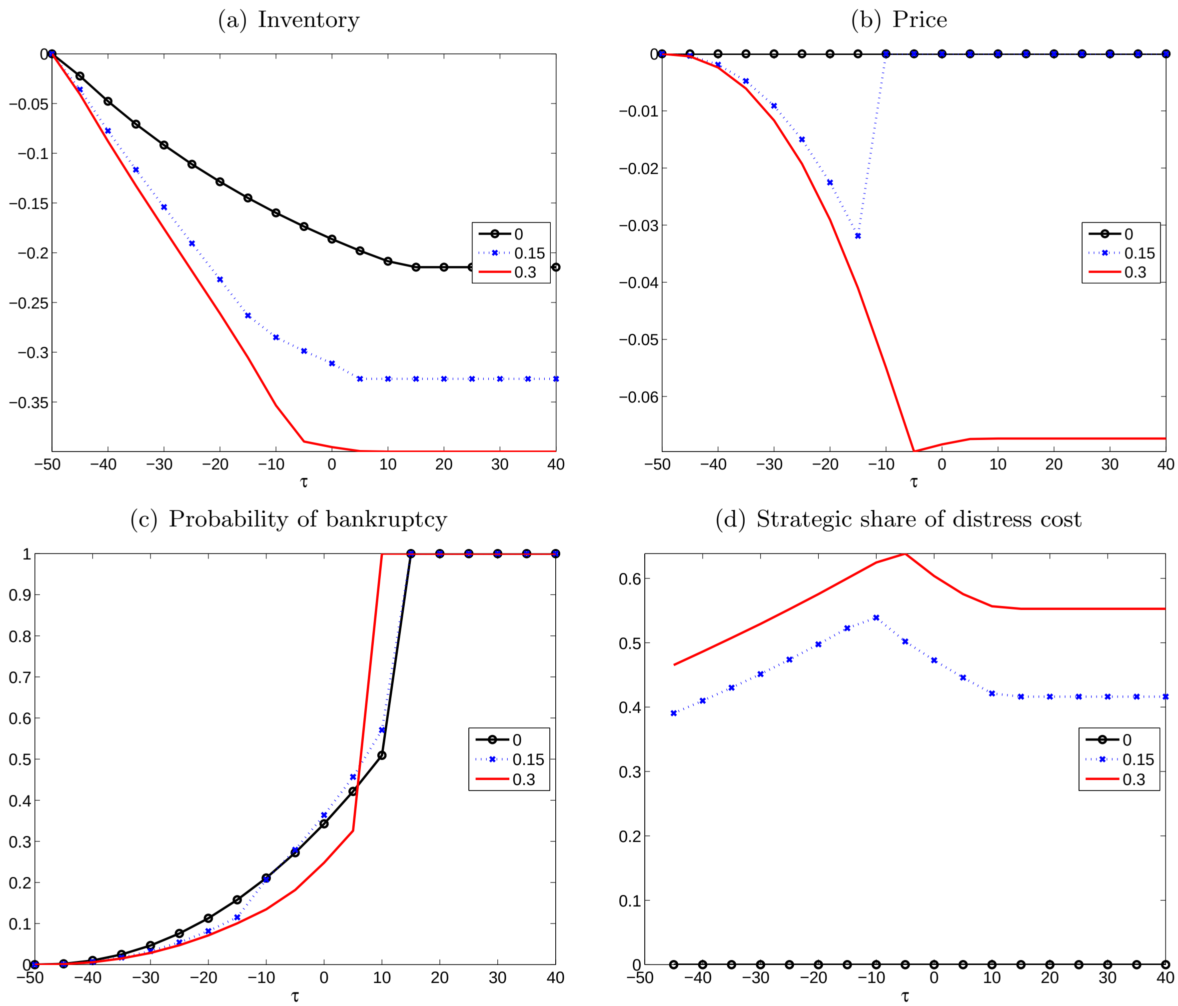
For any $p$, let $Q_m(p) = \arg\max_q \pi(q;p)$. Combining the firm side and the customers side, a $(p,q)$ forms a Rational Expectation Equilibrium if and only if $(p,q)$ satisfy:

$$q = Q_m(p) = Q_b(p).$$

In the presence of multiple REE, the firm will obviously choose the one that leads to the highest profit. This can be achieved as there is a one-to-one mapping between $p$ and $q$. Let the profit be $\Pi_B^*$. Intuitively, this should be lower than the optimal buy-region solution in the main body of the paper.

Next, the firm compares $\Pi_W^*$ and $\Pi_B^*$ and picks the higher one. Intuitively, as $\Pi_W^*$ is the same as in the current version and $\Pi_B^*$ is lower, the ISC region in Figure 4 should expand. To see if our main insights in the paper remain valid, we conduct a set of numerical experiments under the same parameters used in Figure 5. The corresponding results are presented in Figure 12.

**Figure 12 Operational decisions and performance when inventory $q$ is unobservable.**




Notes: $D \sim Triangular(0, 100, 50)$, $v = 1$, $c = 0.6$, $s = 0.5$, $b = 0.3$. Panel 12(a) (12(b)) represents the relative difference between the $q$ ($p$) and the corresponding newsvendor benchmark $q^{NV}$ ($v$).

By comparing Figures 5 and 12, we note that the firm's operational decision and performance follow very similar patterns whether inventory is observable or non-observable to customers, suggesting that our main insights remain unchanged. That said, we can also observe that when inventory $q$ is unobservable, the firm needs to lower inventory and price for a smaller threshold $\tau$. This echoes the findings established in Su and Zhang (2008, 2009) that quantity commitment can mitigate strategic waiting and improve the firm's profit.

## Appendix I: Single class of customers with a discount factor

In this appendix, we modify the model in the main body of the paper by altering the following two assumptions.

1. All first-period customers are forward-looking.

2. First-period customers' second-period valuation is $v_2 \in [b, s]$. For this assumption, note that while the paper studies the interaction between financial distress and strategic consumer behavior, it *emphasizes* more on financial distress by pointing out that strategic consumer behavior can be another source of financial distress. Therefore, we set up the model so that the value of strategic waiting *only* comes from the possible liquidation sale. For this purpose, we confine $v_2 \in [b, s]$ as for $v_2 > s$, customers may also wait even in the absence of financial distress. $v_2$ can be viewed as customers' *patient level.* A higher $v_2$ suggests that customers are very patient, and are hence more likely to wait, keep everything else the same, while a lower $v_2$ weakens the incentive to wait. To an extreme, when $v_2 = b$, customers have no incentive to wait. We can show that the firm's operational decisions and profit under $v_2 = b$ is exactly the same as in Lemma 2, i.e. $\alpha = 0$ in the model of the main body of the paper.

Under this model, we first establish customers' behavior and then study the firm's operational decision.

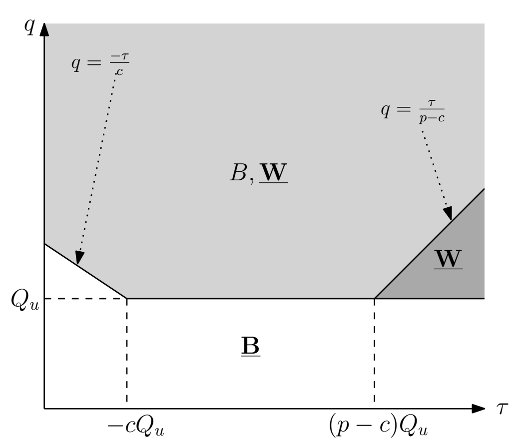
### I.1. Strategic customers' purchase decision under a rational belief.

Similar to Section 4 in the main body of the paper, customers' behavior can be established as follows.

PROPOSITION I.1. Let $Q_u(p) = F^{-1}\left(\frac{v-p}{v_2-b}\right)$, and $Q_{ub}(p) = \max\left(-\frac{\tau}{c},\, Q_u(p)\right)$.

1. A buy-equilibrium exists if and only if $q \geq \frac{\tau}{p-c}$ or $q \leq Q_u(p)$.
2. A wait-equilibrium exists if and only if $q > Q_{ub}(p)$.
3. The wait-equilibrium is more appealing to customers when both equilibria co-exist.

**Figure 13 Strategic customers behavior in equilibrium**




Proposition I.1 is illustrated in Figure 13. Compare this with equilibrium condition as summarized in Proposition 1 (and correspondingly, Figure 3) in the main body of the paper, as shown, the existence conditions for the wait-equilibrium has a similar piece-wise linear structure as the in Figure 3: when the inventory level is sufficiently high, anticipating other customers wait for the liquidation sale, an individual customer also finds it more beneficial to wait. Indeed, by setting $v_s = s$, the existence condition for the wait-equilibrium in this model is the same as the one in the main body of the paper when $\alpha = 1$, i.e. all customers are strategic.

The existence condition for the buy-equilibrium differs from the corresponding result in the main body of the paper. However, the result remain unchanged that when both the buy- and wait-equilibrium co-exist, the wait-equilibrium is more appealing to strategic customers. Therefore, when inducing customers to purchase, the firm has to ensure that $(p,q)$ satisfies $q \le Q_{ub}(p)$, similar to the result in the main body of the paper.

### I.2. Optimal price and inventory decisions.

With an understanding of customers' purchasing behavior, we characterize the firm's optimal price and inventory decisions. Clearly, as all first-period customers are homogenous, the only sensible strategy the firm can adopt is to induce all of them to purchase in the first period, similar to Su and Zhang (2008). Under $(p,q)$ such that all first-period customers purchase, the firm's profit depends on the first-period realized demand $D$ in the following ways.

1. For $D < q$, the firm's first-period revenue is $pD$. In the second period, if $pD - cq \le \tau$, or equivalently, $D \le d_\tau^B := \frac{\tau + cq}{p}$, the firm goes bankruptcy, and the revenue from the liquidation sale is $b(q-D)$. On the other hand, for $D > d_\tau^B$, the firm survives and the revenue from the salvage sale is $s(q-D)$.
2. For $D \ge q$, the inventory is sold out, and hence the firm's profit is $(p-c)q$.

Combining the above scenarios, the firm's total expected profit is:

$$\pi_u = -cq + p\int_{d_l}^{q} x dF(x) + b\int_{d_l}^{d_\tau^B} (q-x)dF(x) + s\int_{d_\tau^B}^{q} (q-x)dF(x) + pq[1-F(q)]. \tag{41}$$

$$= (p-c)q - (p-s)\int_{d_l}^{q} (q-x)dF(x) - (s-b)\int_{d_l}^{d_\tau^B} (q-x)dF(x). \tag{42}$$

Regarding the optimal solution, similar to the main body of the paper, $(p^*, q^*)$ should either be the unconstrained optima, or lay on the buy-wait boundary as illustrated in Figure 13. Therefore, there are three possible candidates.

1. The unconstrained optimal, which is $(v, q^{NV})$. Clearly, this solution satisfies $q \le Q_{ub}(p)$ if and only if $\tau \le -cq^{NV}$.
2. If $(p^*, q^*)$ lay on the downward sloping part of the buy-wait boundary, the optimal solution is clearly $p^* = v$, and $q^* = -\frac{\tau}{c}$. Clearly. The corresponding profit function is:

$$\pi_u^L = -\left(\frac{v}{c} - 1\right)\tau + (v-s)\int_{d_l}^{-\tau/c} \left(-\frac{\tau}{c} - x\right) dF(x). \tag{43}$$

Clearly, $\pi_u^L$ decreases in $\tau$, but it is independent of $v_2$, the customers' patience level.. Also, note that $d_\tau^B = 0$ in this setting. In other word, under $(v, q^*)$, to avoid strategic waiting, the firm completely eliminates the probability of bankruptcy.

3. If $(p^*, q^*)$ lay on the flat part of the buy-wait boundary $(p^*, q^*)$, i.e. $p^* = v - (v_2 - b)F(q^*)$. $(p^*, q^*)$ can be determined by solving the following profit function.

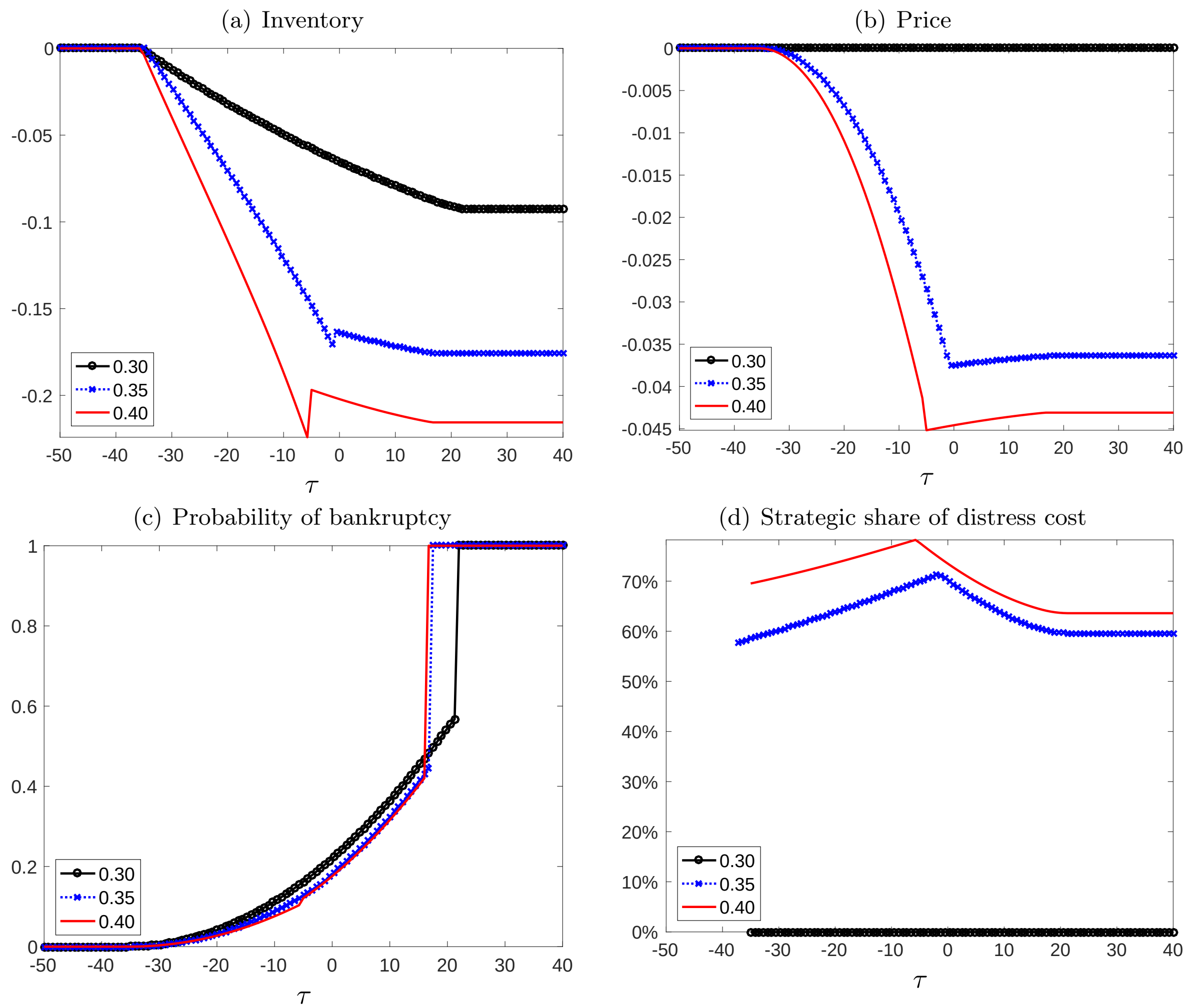
$$\pi_u^H = (p-c)q - (p-s)\int_{d_l}^{q} (q-x)dF(x) - (s-b)\int_{d_l}^{d_\tau^B} (q-x)dF(x). \tag{44}$$

$$= [v - c - (v_2 - b)F(q)]q - [v - s - (v_2 - b)F(q)] \int_{d_l}^{q} (q - x)dF(x) - (s - b) \int_{d_l}^{\frac{cq+\tau}{v-(v_2-b)F(q)}} (q - x)dF(x) \tag{45}$$

In this case, the firm mitigates strategic waiting by directly controlling inventory. It is obvious that at the optimal $(p^*, q^*)$ for this scenario, we have $p^* < v$, i.e. the firm pulls both the inventory and price levers to eliminate strategic waiting. In addition, by the Envelope Theorem, it can also be shown that under the optimal $(p^*, q^*)$, the profit $\pi_u^H$ should decrease in $\tau$ through $d_\tau^B$.

Note that the above three candidate optimal solutions correspond approximately to Region $NV$, $BL$, and $BH$ in Figure 4, suggesting that the single customer class model shares some similarities with the basic model in the paper on the mechanism to that induces customers to purchase.

**Figure 14 Optimal decisions under a single customer class with a discount factor**




Notes: $D \sim Triangular(0, 100, 50)$, $v = 1$, $c = 0.6$, $s = 0.5$, $b = 0.3$. Different lines represent different $v_2$, with the corresponding numbers in the legend. Figure 14(a) and 14(b) plot the relative difference between the corresponding quantities and the newsvendor benchmark $(v, q^{NV})$.

The optimal decision and the corresponding performance is illustrated in Figure 14. As shown in Figure

14(a), with a single class of strategic customers, $q$ in general decreases in $\tau$ and $v_2$, which captures how patient (or equivalently, *strategic*) customers are. This pattern is similar to that under the current model in the main body of the paper (Figure 5(a)). On price (Figure 14(b)), we observe that when customers are more strategic, in general the firm has to offer a deeper price discount, similar to the model in the paper. However, as we argue above, with a single class of strategic customers, the firm always induce customers to purchase. As a result, the optimal price never reverts to $v$ as in the model in the paper. Combining inventory and price, we observe the three stages as discussed above: when $\tau$ is low, the firm simply offers the newsvendor benchmark $(v, q^{NV})$. As $\tau$ increases, the firm focuses on reducing inventory, while keeping price at $v$. Finally, for sufficiently high $\tau$, the firm uses both price and inventory to induce customers to purchase. This pattern is also consistent with the "pecking order" result we highlight in the main body of the paper. Regarding the probability of bankruptcy, as shown in Figure 14(c), similar to Figure 5(c), for low $\tau$, under the optimal decisions, the probability of bankruptcy is lower when customers are more strategic. Finally, Figure 14(d) shows that the strategic share of distress cost is higher when customers are more strategic while the level of financial distress is at a medium level, also similar to the pattern under the current model in the paper (Figure 5(d)).

In summary, the only major difference between the optimal decisions and performance under the single customer class model and those under the current model in the paper is that the firm does not revert its optimal price to $v$ for high $\tau$.

### I.3. Proofs

*Proof of Proposition I.1.* We first proof the existence condition of the two equilibria separately, and then compare the two.

First, on the existence condition of the buy-equilibrium, if $\tau \leq (p-c)q$ and all other customers buy, waiting has no value, and hence the buy-equilibrium exists. If $\tau > (p-c)q$ and all other customers buy, the firm cannot survive, and the expected value of waiting is $(v_2 - b)F(q)$, i.e. the demand is lower than inventory. Therefore, all customers purchase is an equilibrium if $p \leq v - (v_2 - b)F(q)$.

Second, on the existence condition of the wait-equilibrium, if $-cq \geq \tau$, even assuming all other customers wait, waiting has no value for a single customer. So wait-equilibrium does not exist. For $\tau > -cq$, if all other customers wait, the firm cannot survive. Therefore, the expected value of waiting is $(v_2 - b)F(q)$. Therefore, all customers wait is an equilibrium if and only if $p > v - (v_2 - b)F(q)$.

Finally, when both equilibria co-exist, the wait-equilibria are more appealing to customers for the same reason in Proposition 1 and the details are omitted. □

## Appendix J: Maximizing the probability of survival

In this appendix, we examine the firm's optimal decisions with the objective to maximize its survival probability. To guarantee the uniqueness of the solution, when there are multiple solutions that lead to the same probability of survival, we assume that the firm prefers the one that maximizes profit. We focus on the case without deferred discounts, corresponding to Section 5 in the paper.

Clearly, given price and inventory $(p,q)$, strategic customers' behavior remains the same as characterized in Section 4 in the paper. Under such behavior, the firm's optimal decisions $(p^*,q^*)$ and the corresponding probability of bankruptcy is summarized in the following proposition.

PROPOSITION J.1. Let $T_m = \max_{p\le v}(p-c)F^{-1}\left(\frac{v-p}{s-b}\right)$, and $p_T(\tau) = \max_{(p-c)Q_s\ge\tau} p$. When the firm's primary objective is to minimize the probability of bankruptcy,

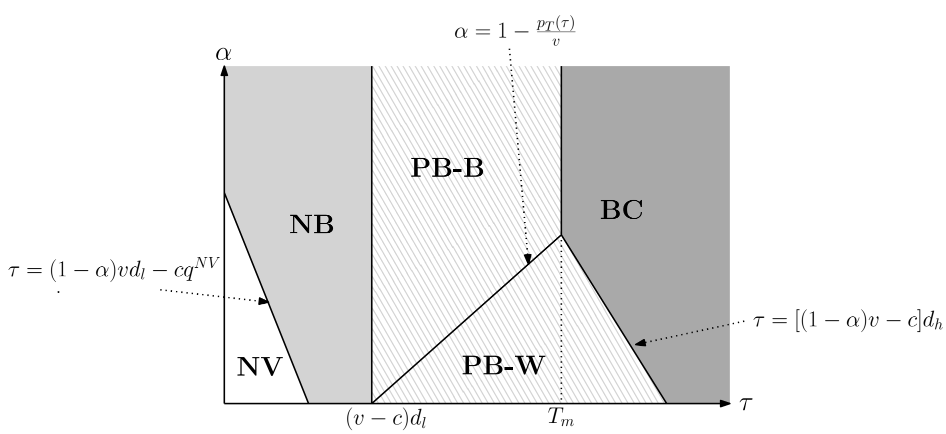
1. for $\tau \le T^D(\alpha)$, the firm's optimal decisions are $(p^*,q^*) = (v, q^{NV})$.
2. for $\tau \in (T^D(\alpha),\,(v-c)d_l)$, the firm's optimal inventory $q^* < q^{NV}$. Under the optimal strategy, the firm's probability of bankruptcy is zero.
3. for $\tau \in ((v-c)d_l,\, \max([(1-\alpha)v-c]d_h, T_m))$, the optimal decisions are:

$$(p^*,\ q^*) = \begin{cases} \left(v,\ \frac{\tau}{(1-\alpha)v-c}\right) & \text{if } \alpha \le 1-\frac{p_T(\tau)}{v}. \\ \left(p_T(\tau),\ \frac{\tau}{p_T(\tau)-c}\right) & \text{otherwise.} \end{cases} \tag{46}$$

and the corresponding probability of bankruptcy is $F(q^*)$.

4. for $\tau \ge \max([(1-\alpha)v-c]d_h,\, T_m)$, the optimal decisions $(p^*,q^*)$ are the same as the profit maximization case, and the firm's probability of bankruptcy is 100%.

**Figure 15 Illustration of the firm's optimal strategies when maximizing the probability of survival**




*Notes.* **NV** represents the region where the *newsvendor* solution is optimal; **NB** for the region with *no bankruptcy* yet the newsvendor solution is not optimal; **PB-B** represents the region where the optimal strategy induces the strategic consumers to *buy* and the resulting probability of bankruptcy is between (0%, 100%); **PB-W** induces the strategic consumers to *wait* and the resulting probability of bankruptcy is between (0%, 100%); **BC** represents the region where the optimal strategy leads to *bankruptcy with certainty.*

The proposition is illustrated in Figure 15. Clearly, when $\tau \le T^D(\alpha)$ as defined in the main body of the paper (Region NV), the firm can avoid bankruptcy even if she stocks the newsvendor quantity and does not offer price discount. As $\tau$ increases (Region NB), the firm can still completely eliminate the probability of bankruptcy, but may need to lower price and/or inventory. In particular, when $\tau = (v-c)d_l$, the firm's optimal decision is $p^* = v$ and $q^* = d_l$, i.e. the firm sets her inventory level to the lowest possible demand realization. It reflects that in this region, to maximize the probability of bankruptcy, the firm behaves very conservatively. Accordingly, the value of the firm suffers from severe *under-investment*.

As $\tau$ continues to increase, it is impossible to completely eliminate bankruptcy. Accordingly, the optimal strategy can be further classified into two categories depending whether strategic customers are induced to buy (Region PB-B) or to wait (Region PB-W). Both regions share two common features. First, the firm can only avoid bankruptcy if and only if all inventory is sold in the first period. For example, when $\alpha$ is small, the optimal wait-region solution is optimal. Under $\tau$, the firm stocks at $\frac{\tau}{(1-\alpha)v-c}$, which is exactly the smallest demand realization that allows the firm to avoid bankruptcy.

Second, in both Region PB-B and PB-W, the amount of inventory $q$ *increases* in the level of financial distress $\tau$. For instance, while $q^* = d_l$ at $\tau = (v-c)d_l$, when $\tau = [(1-\alpha)v - c]d_h$ for small $\alpha$, the optimal inventory level is actually $d_h$. The two features suggest that when bankruptcy cannot be completely eliminated, the firm actually adopt more aggressive operational strategies to reach for survival. Therefore, in this region, the value of the firm may suffer from severe *over-investment*.

Finally, when $\tau$ is sufficiently large (Region BC), bankruptcy is unavoidable even at the highest possible demand realization. Therefore, the firm simply adopts the strategy that maximizes her profit as in the main body of the paper.

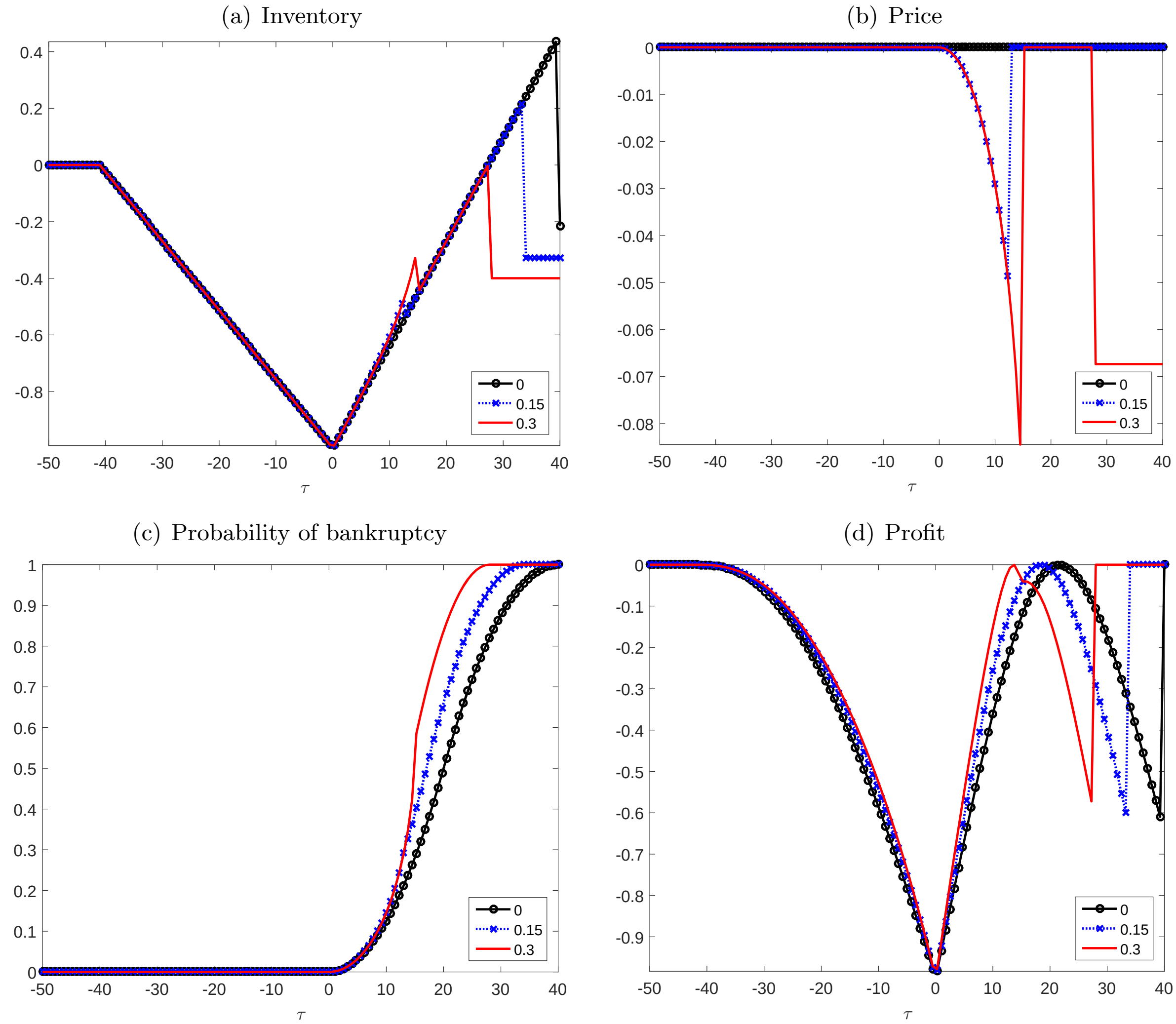
Comparing the above results with the optimal strategy under the objective of profit maximization, we note that there are some similarities between the two strategies. First, when $\tau$ is very small or very large, the two strategies are identical. Second, in general, both strategies induce customers to purchase early when $\alpha$ is sufficiently high. However, the two strategies are also different, most significantly when $\tau$ is in the middle range (Regions PB-B, PB-W). In this region, when the firm minimizes the probability of bankruptcy, she actually order more when her financial distress deepens, while the optimal order quantity decreases in $\tau$ under the objective of profit maximization as in the paper. Although we do not have direct evidence in which objective managers adopt in practice, the result under profit maximization seems to be more consistent with empirical evidence that firms often hold lower level of inventory under financial distress (Chevalier 1995, Matsa 2011).

The above results are further illustrated in Figure 16 based on the same set of parameters used in Figure 5. The non-monotonicity of inventory level $q$, as shown in Figure 16(a), is particularly notable. Observe that at $\tau = 0$, to completely eliminate the probability of bankruptcy, the firm simply does not order *anything*. However, for sufficiently large $\tau$, the firm may order *above* the newsvendor level. Such behavior is reflected in Figure 16(d), which shows that the firm sacrifices most value around these extreme inventory levels.

### J.1. Proofs

*Proof of Proposition J.1.* We prove this proposition by three steps.

**Figure 16 The impact of using survival maximization as the objective.**




Notes: $D \sim Triangular(0, 100, 50)$, $v = 1$, $c = 0.6$, $s = 0.5$, $b = 0.3$. Figure 16(a) shows the relative difference between the optimal inventory and the newsvendor benchmark $q^{NV}$. Figure 16(d) shows the relative difference between the optimal profit under the survival maximization objective and that under the value/profit maximization objective.

1. We show that the firm can guarantee survival if and only if $\tau \leq (v - c)d_l$. For the sufficient part, note that for any $\tau$ in this range, it is easy to show that under $p = v$ and $q = d_l$, all customers purchase, and the firm never goes bankrupt. For the necessary part, note that the maximal guaranteed revenue for the firm is $vd_l$. Also, if the firm orders less than $d_l$, the firm's minimal profit can be improved if the firm orders more. Therefore, if $\tau > (v - c)d_l$, the firm will always go bankrupt with a positive probability.

In this region, as the realized bankruptcy probability is zero, customers have no incentive to wait, and hence we only need to focus on the buy-region. When $\tau \leq (1 - \alpha)vd_l - cq^{NV}$, $(v,\, q^{NV})$ leads to zero bankruptcy probability, and hence the solution is optimal under the objective of the survival maximization.

2. We show that for $\tau \in ((v - c)d_l, \max([(1 - \alpha)v - c]d_h, T_m)$, the optimal strategy is as stated in Statement 2 in the proposition.

In this region, to identify the solution that minimizes probability of bankruptcy, we consider two scenarios: the optimal solution in the buy-region, and the optimal solution in the wait-region.

(a) Consider the wait-region solution first. Clearly, the price is still $v$, and the probability of bankruptcy under $q \in [d_l,\, d_h]$ is $F\left(\frac{cq+\tau}{(1-\alpha)v}\right)$ if $\frac{cq+\tau}{(1-\alpha)v} \leq q$, and 100% otherwise. Therefore, the probability of bankruptcy is minimized at the boundary, i.e. $q = \frac{cq+\tau}{(1-\alpha)v}$, or equivalently, $q^* = \frac{\tau}{(1-\alpha)v-c}$, and the probability of bankruptcy is $F\left(\frac{\tau}{(1-\alpha)v-c}\right)$. For $\tau > [(1-\alpha)v - c]d_h$, there exists no solution in the wait-region that can avoid bankruptcy.

(b) Consider the buy-region solution. Similarly, under $(p,q) \in \Omega^B$, the probability of bankruptcy is $F\left(\frac{cq+\tau}{p}\right)$ if $\frac{cq+\tau}{p} \leq q$, and 100% otherwise. Therefore, the optimal strategy solves for the following optimization problem:

$$\min_{p,\,q} \quad \frac{cq+\tau}{p} \tag{47}$$

$$\text{s.t.} \quad (p-c)q \geq \tau \tag{48}$$

$$q \leq \max\left(Q_s, \frac{(1-\alpha)pQ_s - \tau}{c}\right) \tag{49}$$

Note that given fixed $p$, the objective function decreases in $q$, and therefore the second constraint is irrelevant (unless there exists no feasible solution). Therefore, the optimal $q^* = \frac{\tau}{p-c}$, and hence the above optimization problem is simplified to maximize $p$ subject to the constraint $(p-c)\max\left(Q_s, \frac{(1-\alpha)pQ_s-\tau}{c}\right) \geq \tau$. Examining the constraint, we note that $Q_s \leq \frac{(1-\alpha)pQ_s-\tau}{c}$ if and only if $[(1-\alpha)p - c]Q_s \geq \tau$. Under this condition, the constraint, $(p-c)\frac{(1-\alpha)pQ_s-\tau}{c} \geq \tau$, is automatically satisfied. On the other hand, for $[(1-\alpha)p - c]Q_s < \tau$, the constraint can be simplified to $(p-c)Q_s \geq \tau$. Combining the above two scenarios, the constraint can be simplified to $(p-c)Q_s \geq \tau$. Clearly, for $\tau > T_m$, the constraint is infeasible. For $\tau \leq T_m$, the optimal price in the buy-Region is $p_T(\tau)$ as defined. Furthermore, it is clear that $p_T(\tau)$ decreases in $\tau$.

Combining the optimal solution in the buy-region and that in the wait-region, we find that for $\tau \in ((v-c)d_l, \max([(1-\alpha)v - c]d_h, T_m)$, under the optimal solution, the probability of bankruptcy is $F\left(\min\left(\frac{\tau}{(1-\alpha)v-c},\, \frac{\tau}{p_T(\tau)-c}\right)\right)$, which is between 0% and 100% (not inclusive).

3. when $\tau \geq \max\left([(1-\alpha)v - c]d_h,\, T_m\right)$, it is impossible to avoid bankruptcy, hence the optimal solution is the same as the profit maximization case, and the firm's probability of bankruptcy is 100%. □