Online Supplements to

The Supply Chain Effects of Bankruptcy

Management Science

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Appendix C:

To test the robustness of the results presented in the main body of the paper, we conduct additional analysis in this section. Specifically, we study the implication of the following three assumptions:

1. the cost of reorganization depends on the firm's asset shortfall;
2. the realized cost of reorganization can be observed by all parties;
3. two downstream firms compete in a differentiated Cournot model.

C.1. The cost of reorganization depends on the firm's asset shortfall

In the main body of the paper, we assume the distribution of $\tilde{C}_r$ only depends on the sign of asset shortfall in bankruptcy $D=-(\pi_1^d+\alpha+A+\pi_c^d)$. Consequently, each party's second-period profit is a step function of D: it equals to constant π_c^j if $D\le 0$, and constant π_b^j if $D>0$ as the distressed firm files for bankruptcy. In this section, we relax this assumption and assume that $\tilde{C}_r$ (stochastically) increases in D. We focus on the endogenous differentiated pricing case (Sections 5 and 6 in the main body of the paper), but similar results can be shown for the other two scenarios.

As $\tilde{C}_r$ increases (stochastically) in D for $D>0$, according to the definition of D, it is equivalent that $\tilde{C}_r$ decreases (stochastically) in α and π_1^d for $\alpha\le\alpha_b$, where as defined in the main body in the paper, $\alpha_b=-A-\pi_1^d-\pi_c^d$ is the bankruptcy threshold. Therefore, we can write the non-distressed firm and the supplier's expected profit in bankruptcy conditional on π_1^d and the realized shock α are: $\pi_b^i=\Phi\pi_r^i+(1-\Phi)\pi_l^i$. As Φ, π_r^i and π_l^i are determined by the distribution of the cost of reorganization $\tilde{C}_r$, Φ, π_r^i and π_l^i are all functions of the realized liquidity shock α and the first-period profit π_1^d. To capture the dependency between π_b^i and π_1^d and α, we write $\pi_b^i(\alpha,\pi_1^d)$. The magnitude and monotonicity of these quantities are still governed by Proposition 2 and Corollaries 1, only that the cost of reorganization $\tilde{C}_r$ is now parameterized by α and π_1^d through D. Indeed, one can actually view how profits changes as D increases by re-interpreting the x-axes in Figure 3 as the amount of asset shortfall.

Move to the first period. Before the realization of $\tilde{\alpha}$, the unconditional second-period profit of the non-distressed firm and the suppliers are:

$$\pi_2^i=\int_{-\infty}^{\alpha_b}\pi_b^i(\alpha,\pi_1^d)dF(\alpha)+[1-F(\alpha_b)]\pi_c^i,\quad i=n,s. \tag{17}$$

By integrating π_b^i over $\tilde{\alpha}$, π_2^i is only a function of π_1^d. Note that as shown in Sections 6 and 7, the three supply chain effects we highlight in the paper are quantitatively related to the (marginal) competitor impact (δ^n), the (marginal) supplier impact (δ^s), and the second-order competitor impact (ϵ^n). We start with the bail-out effect and the corresponding supplier impact. As defined in Section 5,

$$\delta^s=-\frac{\partial\pi_2^s}{\partial\pi_1^d}=-\int_{-\infty}^{\alpha_b}\frac{\partial\pi_b^s}{\partial\pi_1^d}dF(\alpha)+f(\alpha_b)\left[\pi_b^s(\alpha_b,\pi_1^d)-\pi_c^s\right]. \tag{18}$$

As a special case, in the main body of the paper, as π_b^s is constant for $\alpha\le\alpha_b$, $\int_{-\infty}^{\alpha_b}\frac{\partial\pi_b^s}{\partial\pi_1^d}dF(\alpha)=0$. However, by re-interpreting Corollary 1, we can show that as $\tilde{C}_r$ increases stochastically in D according to the Monotone Likelihood Ratio Property, $\frac{\partial\pi_b^s}{\partial\pi_1^d}>0$ for $\alpha\le\alpha_b$. Combining this with the fact that $\pi_b^s(\alpha_b,\pi_1^d)\le\pi_c^s$, we have $\delta^s<0$. That is, the supplier always has the incentive to bail out the distressed firm, consistent with the main body of the paper.

Similarly, for the competitor impact δ^n,

$$\delta^n = -\frac{\partial \pi_2^n}{\partial \pi_1^d} = -\int_{-\infty}^{\alpha_b} \frac{\partial \pi_b^n}{\partial \pi_1^d} dF(\alpha) + f(\alpha_b)\left[\pi_b^n(\alpha_b, \pi_1^d) - \pi_c^n\right]. \tag{19}$$

For $\alpha < \alpha_b$, as π_1^d decreases, the cost of reorganization $\tilde{C}_r$ increases (stochastically). Therefore, according to Figure 3(b), as π_1^d decreases, π_b^n first decreases and then increases, and hence, $\frac{\partial \pi_b^n}{\partial \pi_1^d}$ is positive when α is marginally smaller than α_b, and negative when α is very small. The aggregate effect, as reflected by the integration, depends on the distribution of $\tilde{\alpha}$. As $\pi_b^n(\alpha_b, \pi_1^d)$ could also be greater or less than π_c^n, δ^n could be positive or negative. Again, this is consistent with the main body of the paper that the distressed firm's bankruptcy could be a positive or negative externality to its non-distressed competitor.

Finally, regarding the second-order competitor effect (ϵ^n), we have:

$$\frac{\epsilon^n}{\pi_1^d} = -\frac{\partial \delta^n}{\partial \pi_1^d} = \int_{-\infty}^{\alpha_b} \frac{\partial^2 \pi_b^n}{\partial (\pi_1^d)^2} dF(\alpha) - f(\alpha_b)\left(\frac{\partial \pi_b^n(\alpha_b, \pi_1^d)}{\partial \pi_1^d}\right) + f'(\alpha_b)\left[\pi_b^n(\alpha_b, \pi_1^d) - \pi_c^n\right]. \tag{20}$$

As observed in Figure 3(b), π_b^n is convex on the mean of the reorganization cost. Under the assumption that the mean of the reorganization cost linearly increases in D, and hence π_1^d, we have $\frac{\partial^2 \pi_b^n}{\partial (\pi_1^d)^2} > 0$. The second and third terms, however, can be both positive or negative. Therefore, the aggregate effect may also be positive or negative, similar to the main body of the paper.

As discussed in the main body of the paper, the supplier's choice of optimal first-period wholesale prices is heavily dependent on the sign and (relative) magnitude of δ^s, δ^n and ϵ, which exhibit similar properties whether we assume $\tilde{C}_r$ depends on D or not. Therefore, we conclude that the qualitative structure of the firms' behavior and profitability remain unchanged.

C.2. The realized cost of reorganization can be observed by all parties

In the main body of the paper, the assumption related to the cost of reorganization consists of two parts: first, the cost of reorganization is random, and the distribution of this random cost is public knowledge; second, the realized cost of bankruptcy is only revealed to the management of the bankrupt firm *after* the firm files for bankruptcy. In this section, we relax the second part of the assumption by assuming that all parties observe the realized cost of reorganization C_r. We focus on endogenous differentiated pricing case. We first show its impact in the post-bankruptcy decisions, and then investigate its impact on the pre-bankruptcy decisions.

Assuming the distressed firm is in bankruptcy, and the *realized* cost of bankruptcy is C_r. When offering w_r^d and w_r^n, the supplier compares its profit under reorganization against under liquidation. Parallel to Proposition 2, its optimal choice of wholesale prices in reorganization is summarized as follows.

Proposition C.1 *In bankruptcy, the supplier offers $w_r^n = \frac{\mu_2}{2}$ to the non-distressed firm. The bankrupt firm's wholesale price and the reorganization outcome depends on the realized cost of reorganization C_r,*

1. *For $C_r \in [0, c_r^L]$, $w_r^d = \frac{\mu_2}{2}$, and the bankrupt firm is reorganized.*
2. *For $C_r \in (c_r^L, c_r^U]$, the supplier offers $w_r^d = \frac{3\mu_2}{4} - \frac{3\sqrt{C_r}}{2}$, and the bankrupt firm is reorganized.*
3. *For $C_r > c_r^U$, the bankrupt firm is liquidated.*

where $c_r^L = \frac{\mu_2^2}{36}$ and $c_r^U = \frac{\mu_2^2}{9}$.

Proof of Proposition C.1. If the bankrupt firm is liquidated, the supplier offers $w_l^n = \frac{\mu_2}{2}$ to the non-distressed firm, receives quantity $q_l^n = \frac{\mu_2}{4}$, and realizes profit $\pi_l^s = \frac{\mu_2^2}{8}$. When the supplier tries to help the bankrupt firm reorganize, under w_r^n and w_r^d, the two downstream firms' ordering quantities are $q_r^d = \frac{1}{3}(\mu_2 - 2w_r^d + w_r^n)$ and $q_r^n = \frac{1}{3}(\mu_2 - 2w_r^n + w_r^d)$, with corresponding profits $\pi_r^d = \left(\frac{\mu_2 - 2w_r^d + w_r^n}{3}\right)^2$ and $\pi_r^n = \left(\frac{\mu_2 - 2w_r^n + w_r^d}{3}\right)^2$. Facing these responses, the supplier maximizes its profit π_r^s, which follows:

$$\pi_r^s = \frac{\mu_2(w_r^d + w_r^n) - 2(w_r^d)^2 - 2(w_r^n)^2 + 2w_r^d w_r^n}{3}. \tag{21}$$

subject to the constraint $\pi_r^d \geq C_r$. Obviously, when the constraint is not binding, the supplier solves the unconstrained problem, leading to: $w_r^d = w_r^n = \frac{\mu_2}{2}$, and hence $q_r^d = q_r^n = \frac{\mu_2}{6}$ and $\pi_r^d = \pi_r^n = \frac{\mu_2^2}{36}$.

On the other hand, when the constraint is binding, that is, $\pi_r^d = C_r$. Replacing $\pi_r^d = C_r$ into the quantity response, the constraint becomes $w_r^n = 2w_r^d + 3\sqrt{C_r} - \mu_2$. Substituting this to the supplier's profit function and take the derivative with respect to w_r^d, we have:

$$\frac{d\pi_r^s}{dw_r^d} = \mu_2 - 2(2w_r^d + 3\sqrt{C_r} - \mu_2). \tag{22}$$

Setting $\frac{d\pi_r^s}{dw_r^d} = 0$ leads to $w_r^d = \frac{3\mu_2 - 6\sqrt{C_r}}{4}$, and $w_r^n = \frac{\mu_2}{2}$, resulting $q_r^n = \frac{\mu_2 - 2\sqrt{C_r}}{4}$, $q_r^d = \sqrt{C_r}$, the market price is $p_r = \frac{3\mu_2 - 2\sqrt{C_r}}{4}$. $\pi_r^n = \frac{(\mu_2 - 2\sqrt{C_r})^2}{16}$, and finally, $\pi_r^s = \frac{\mu_2^2}{8} + \frac{\mu_2\sqrt{C_r}}{2} - \frac{3C_r}{2}$. Therefore, $\pi_r^s \geq \pi_l^s$ if and only if $C_r \leq \frac{\mu_2^2}{9}$. □

The above result makes intuitive sense. As the realized cost of reorganization can be observed by the supplier, when the realized cost is small, the supplier does not grant any concession; as the cost increases, the bankrupt firm receives a lower wholesale price so that the resulting profit is only sufficient to cover the realized cost. In the region, the non-distressed firm suffers. Finally, as the cost further increases, it is too costly for the supplier to bail out the bankrupt firm, hence it prefers to let the bankrupt firm be liquidated. Therefore, the bankrupt firm's *expected* profit in bankruptcy (taken the cost of reorganization into consideration) is $\pi_b^d = \int_0^{c_r^L} (c_r^L - x) dG(x)$. Comparing with the result in the main body of the paper, not surprisingly, the bankrupt firm's profit is lower as it can not extract any information rent. This profit is always lower than that without bankruptcy. However, the probability of successful reorganization is higher comparing to the case where C_r is only observable to the distressed firm.

On the other hand, the supplier's profit conditional on bankruptcy is:

$$\pi_b^s = G(c_r^L)\left(\frac{\mu_2^2}{6}\right) + \int_{c_r^L}^{c_r^U} \left(\frac{\mu_2^2}{8} - \frac{3x - \mu_2\sqrt{x}}{2}\right) dG(x) + [1 - G(c_r^U)]\left(\frac{\mu_2^2}{8}\right). \tag{23}$$

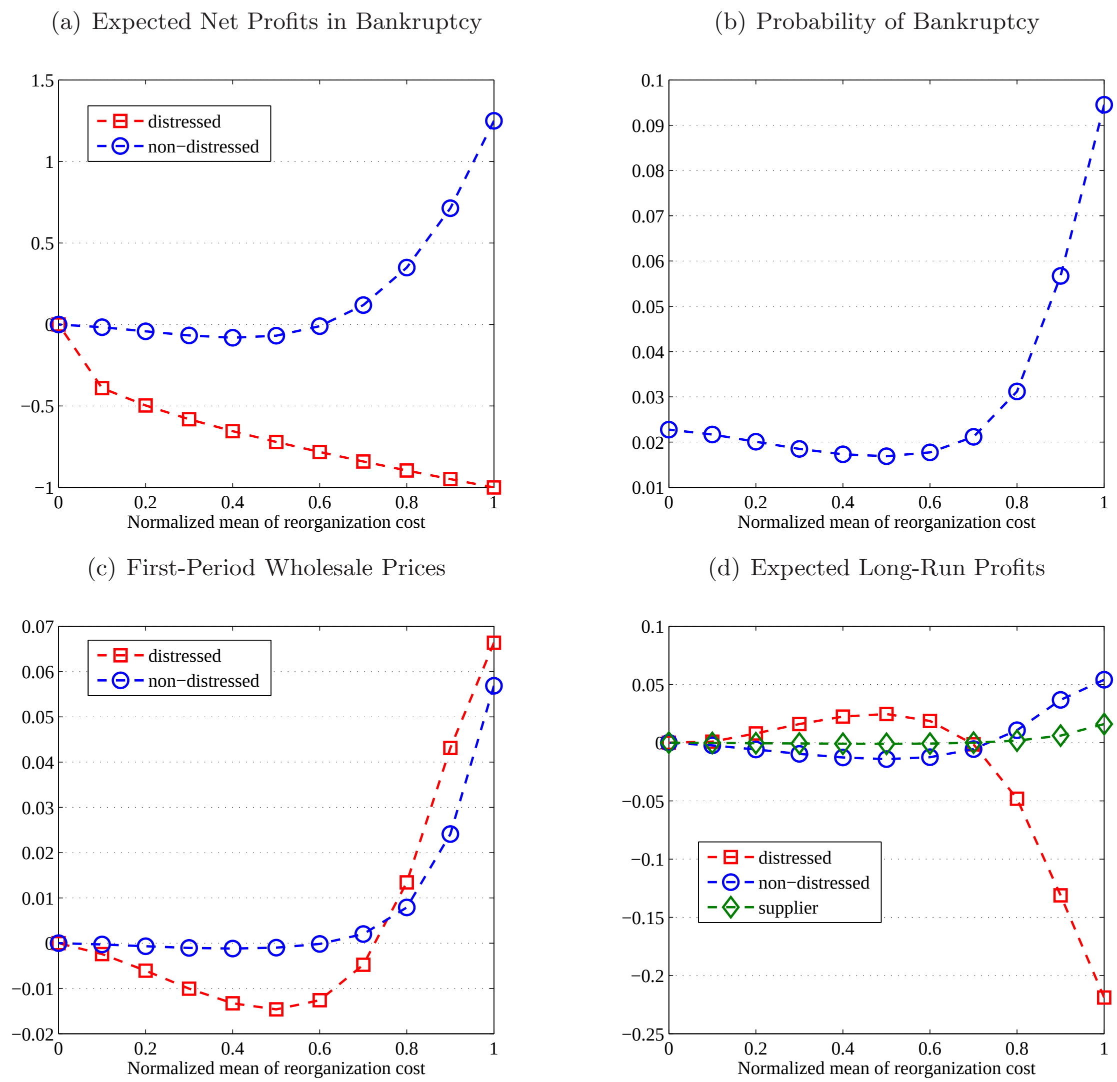
Compared to the case with unobservable C_r, the supplier is always better off knowing the realized cost of reorganization. This implies that the bail-out effect in the first period will be weaker, hurting the distressed firm and benefiting the non-distressed competitor.

Finally, the non-distressed firm's profit conditional on bankruptcy is:

$$\pi_b^n = \frac{\mu_2^2}{36} + [1 - G(c_r^L)]\left(\frac{5\mu_2^2}{144}\right) - \int_{c_r^L}^{c_r^U} \left(\frac{\mu_2\sqrt{x} - x}{4}\right) dG(x). \tag{24}$$

It is easy to see that when $C_r \geq c_r^U$, the non-distressed firm enjoys a monopoly position in both cases. However, as C_r decreases, the effect is mixed. On the one hand, when C_r is relatively small, the non-distressed firm benefits from the information as the supplier is granting less concession to the distressed firm. However, for a larger C_r, the non-distressed firm is hurt as the the supplier offers greater concessions to keep the distressed firm alive. The aggregated effects hence depend on the shape of $G(\cdot)$. Similarly, the effect of information structure on consumer surplus is also mixed. To quantify this impact, we conduct numerical experiments under the same parameters as in Sections 5 and 6. As shown in Figure 8(b), comparing with Figure 3(b), as we conjectured earlier, the non-distressed firm's profit in bankruptcy is higher than the scenario with unobservable reorganization cost when the mean of reorganization cost is small, but lower when the mean cost is large. However, its magnitude relative to the no-distress benchmark is preserved.

Figure 8 Numerical Results with Observable Cost of Reorganization under Endogenous Differentiated Pricing



Notes. Parameters used are the same as in Figure 5. y-axis presented as the relative differences from the no-distress benchmark.

Regarding the implication of observable cost of reorganization in the first period, it is obvious that Proposition 5 and all explanations in Section 6 remain unchanged. The only differences are the magnitude of δ^n,

δ^s, and ϵ^n. As shown in Figures 8(b) to 8(d), the results are similar to that in Figures 4(b), 4(a), and 5(a). Therefore, we conclude that assuming that the cost of reorganization is observable to all parties does not change our main insights qualitatively. Similar results can be shown for the passive supply and endogenous uniform pricing cases.

C.3. Two downstream firms compete in a differentiated Cournot model

In the paper, we assume that the two downstream firms' products are perfect substitutes. In this section, we relax this assumption by modeling the two products as imperfect substitutes as in Singh and Vives (1984). Specifically, we consider a special symmetric case in which the demand functions in the first period are:

$$p_1^d = \mu_1 - q_1^d - \beta q_1^n, \quad p_1^n = \mu_1 - q_1^n - \beta q_1^d, \tag{25}$$

Similarly, in the second period, for $i = c, r, l$, $p_i^d = \mu_2 - q_i^d - \beta q_i^n$ and $p_i^n = \mu_2 - q_i^n - \beta q_i^d$. $\beta \in [0,1]$ in the model captures how close the two products are substitutes. When $\beta = 1$, this model degenerates to the homogenous goods model in the main body of the paper.

With these demand functions, we first establish the no-distress benchmark similar to Section 3.3. Again, we use the subscript a to represent first-period quantities under this benchmark. It is easy to see that without financial distress, the two periods can be decomposed.

For the downstream firms, facing w_1^d and w_1^n in the first period, the equilibrium quantities are:

$$q_a^d = \frac{(2-\beta)\mu_1 + \beta w_1^n - 2w_1^d}{4-\beta^2}; \quad q_a^n = \frac{(2-\beta)\mu_1 + \beta w_1^d - 2w_1^n}{4-\beta^2}. \tag{26}$$

Similarly, in the second period,

$$q_c^d = \frac{(2-\beta)\mu_2 + \beta w_c^n - 2w_c^d}{4-\beta^2}; \quad q_c^n = \frac{(2-\beta)\mu_1 + \beta w_c^d - 2w_c^n}{4-\beta^2}. \tag{27}$$

For the endogenous pricing scenario, it is easy to see that $w_a^d = w_a^n = \frac{\mu_1}{2}$ in the first period, leading to $q_a^d = q_a^n = \frac{\mu_1}{4+\beta}$, profits $\pi_a^d = \pi_a^n = \frac{\mu_1^2}{(4+\beta)^2}$, and $\pi_a^s = \frac{\mu_1}{4+2\beta}$. Not surprisingly, with $\beta = 1$, these quantities degenerate to the corresponding quantities in Section 3.3.

C.3.1. The predation effect. Similar to Section 4, for $i = c, r$, the equilibrium production quantities are:

$$q_r^d = q_c^d = \frac{(2-\beta)\mu_2 + \beta w^n - 2w^d}{4-\beta^2}; \quad q_r^n = q_c^n = \frac{(2-\beta)\mu_2 + \beta w^d - 2w^n}{4-\beta^2}. \tag{28}$$

And $\pi_r^d = \pi_c^d = (q_c^d)^2$, $\pi_r^n = \pi_c^n = (q_c^n)^2$. In liquidation, $\pi_l^n = \frac{1}{4}(\mu_2 - w^n)^2$ and $\pi_l^d = 0$. Again, we define the (marginal) competitor impact $\delta^n = -\frac{\partial \pi_2^n}{\partial \pi_1^d}$, and it follows $\delta^n = f(\alpha_b)\Phi(\pi_l^n - \pi_c^n) \geq 0$, where the equality only holds when $\beta = 0$, $\pi_l^n = \pi_c^n$. This is not surprising as the two firms' demand are independent when $\beta = 0$, and hence the bankruptcy of the distressed firm has no impact on the non-distressed firm. In this sense, the differentiated Cournot case can be seen as a transition from the perfect substitute case in the paper and an independent demand case in which the predation effect is absent.

Move to the first period, the two firms' best-response functions are:

$$q_1^d = \frac{\mu_1 - w^d - \beta q_1^d}{2}; \quad q_1^n = \frac{\mu_1 - w^n - (\beta - \delta^n) q_1^d}{2}. \tag{29}$$

which is the same as in Section 4. Combining the two best-response functions leads to the following results.

Proposition C.2 *The equilibrium quantities are:*

$$q_1^d = q_a^d - \frac{\beta\delta^n}{4-\beta^2+\beta\delta^n} q_a^d; \quad q_1^n = q_a^n + \frac{2\delta^n}{4-\beta^2+\beta\delta^n} q_a^d \tag{30}$$

where δ^n and π_1^d are jointly determined by the following equation.

$$\left(1+\frac{\beta\delta^n}{4-\beta^2}\right)^2 \pi_1^d = \pi_a^d. \tag{31}$$

The proof is similar to Proposition B.1 and the detail is omitted here. This result is consistent with Proposition B.1, confirming that under the differentiated Cournot model, the predation effect still reduces the distressed firm's first-period quantity and increases the non-distressed firm's quantity and the aggregated quantity $(q_1^d + q_1^n)$. Further analysis shows that Corollaries B.1 and B.2 also remain valid structurally. For example, the predation effect could still increase the non-distressed firm's first-period profit π_1^n.

C.3.2. The bail-out effect. To see how the differentiated Cournot model changes the supplier's behavior, we focus on the bankruptcy period (Section 5 in the main body of the paper). When the distressed firm is in bankruptcy, it is easy to see that if the distressed firm is liquidated, the supplier should offer $w_l^n = \frac{\mu_2}{2}$ to the non-distressed firm, leading to $\pi_l^s = \frac{\mu_2^2}{8}$. Under reorganization, the supplier's profit is $\pi_r^s = w_r^d q_r^d + w_r^n q_r^n$. Similar to Proposition 2, the supplier chooses w_r^d and w_r^n to maximize $G(\pi_r^d)(\pi_r^s - \pi_l^s)$.

Proposition C.3 *In bankruptcy, the supplier offers:*

$$w_r^d = \left(1-\frac{\beta}{4}\right)\mu_2 - \left(2-\frac{\beta^2}{2}\right)\sqrt{x^*}, \quad w_r^n = \frac{\mu_2}{2}. \tag{32}$$

where $x^* = \arg\max_{x\in\left[\mu_2^2/(4+2\beta)^2,\mu_2^2/(2+\beta)^2\right)} G(x)[\mu_2\sqrt{x} - (2+\beta)x]$. *When* $G(x)$ *is concave for* $x \in \left[\frac{\mu_2^2}{4(2+\beta)^2}, \frac{\mu_2^2}{(2+\beta)^2}\right]$,

$$x^* = \frac{\mu_2^2}{(4+2\beta)^2}\left[1+\frac{x^* g(x^*)}{G(x^*)+x^* g(x^*)}\right]^2. \tag{33}$$

The proof is similar to that of Proposition 2 and the detail is omitted. Again, the results confirm that under the differentiated Cournot model, the main results in Section 5 remains unchanged: the bail-out effect does not change the non-distressed firm's wholesale price directly, that is, $w_r^n = \frac{\mu_2}{2} = w_l^n = w_c^n$; the distressed firm receives a lower wholesale price $(w_r^d \le w_c^d)$, leading to a operational disadvantage to the non-distressed firm. Therefore, the bail-out effect still acts as a negative externality to the non-distressed firm.

C.3.3. Concluding remarks. As shown above, the predation and bail-out effects play the same roles under the differentiated duopoly model as in the homogenous duopoly model used in the main body of the paper. Further, as the abetment effect originates from the predation effect, it is logical to expect that allowing differentiated products does not have any material impact on this effect and the interplay of the three supply chain effects. Admittedly, we can obtain additional insights related to the product substitutability. However, we feel these insights are not central to the focus of the paper and hence we decide to keep the homogenous duopoly model in the paper.