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Social Responsibility Auditing in Supply Chain Networks

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Abstract. We study a buyer's problem of auditing suppliers within an existing network to ensure social responsibility compliance. The buyer suffers economic damages if a violation at a supplier is exposed (whether by the media, regulator, or nongovernmental organization). To avoid damages, the buyer may audit the network to identify noncompliance. If a supplier fails an audit, the buyer must take one of two costly actions: either rectify the supplier or drop the supplier (along with any dependent suppliers). Dropping a supplier changes the network topology, reducing competition and thereby increasing the buyer's input cost arising from an equilibrium. We show that the buyer's optimal dynamic auditing policy has two subphases: the buyer will first audit and drop some suppliers before either auditing and rectifying all remaining suppliers or halting auditing altogether. By halting, the buyer tolerates some noncompliance in the network ("see no evil, hear no evil"). Within the audit-and-drop subphase, when auditing only in the upper tier, the buyer always audits a *least valuable unaudited supplier*, yielding greater balance in the network. When the buyer audits both tiers, it might choose a supplier other than the least valuable. The buyer may choose a supplier in a pivotal position to help ascertain the viability of a portion of the network ("litmus test"). In extensions, we find that when violations in tier 1 carry a higher penalty for the buyer, the buyer may audit and rectify only tier 1 suppliers; when audits may be inaccurate, the buyer more likely tolerates a greater level of noncompliance.

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1. Introduction

Violations of social responsibility norms by suppliers are widespread. Such violations appear in domains as varied as infringement on human rights (Segal 2019, Teixeira 2019), animal abuse (Phillips 2016, Elejalde-Ruiz 2019), and environmental harm (Rana 2018). A common trait of these examples is that they are *process compliance* violations, which involve the production processes and typically require onsite vetting to detect. Many companies have taken proactive steps to audit their suppliers to ensure compliance on social responsibility. Apple, IKEA, and Nike all publish reports annually documenting their audits (Apple Inc. 2018b, Inter IKEA Group 2019, Nike, Inc. 2020). Such powerful buyers conduct audits throughout the supply network, because their prominence makes them particularly vulnerable to the consumer backlash associated with the exposure of a social responsibility violation.

Many powerful buyers audit not only direct suppliers but also indirect suppliers further upstream. In IKEA's Sustainability Report for 2018, the company identifies 201 upper-tier suppliers that "provides components and materials to IKEA suppliers and sub-suppliers" for verification and audits (Inter IKEA Group 2019, p. 47). Inditex subjects every firm in its supply chain to periodic social audits (Inditex 2020a). Target requires all merchandise suppliers, all the way back to raw materials suppliers, to participate in its responsible sourcing program; the retailer devises monthly audit plans to identify factories to audit (Target Brands, Inc. 2020). Motivated by examples such as these, we consider an auditing plan for a single cycle (e.g., a year) where direct and indirect suppliers are audited. This plan may be considered a part of a buyer's ongoing auditing effort (over multiple years), although we isolate consideration to a single cycle. We consider a buyer auditing its existing supply network: the buyer begins the auditing cycle with the network as left from the previous auditing cycle.

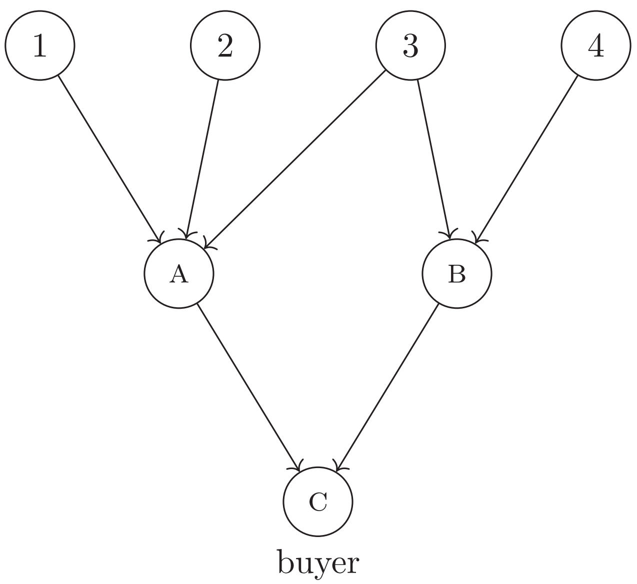
As with other global buyers, Apple Inc. regularly audits suppliers and documents the audits and remedial actions in its annual supplier responsibility progress report (Apple Inc. 2018b). In its 2018 report, Apple detected debt-bonded labor (Apple Inc. 2018b), which the United Nations deems a form of modern slavery (Zeldin 2016). In each case, Apple put the supplier on immediate probation, requiring it to undergo *rectification*, including financial remedies for every affected employee. Another area covered by Apple's report is the sourcing of minerals that may originate from mines that finance armed conflicts (Zhang et al. 2020). Apple separately reported that it *dropped* 10 smelters and refiners from its supply chain in 2017 for noncompliance on the issue of conflict minerals (Apple Inc. 2018a). Other buyers also decide to *rectify* or to *drop* a supplier when an audit uncovers a violation. In 2018, IKEA suspended a supplier in China found to be in violation of IKEA's forestry sustainability requirements (Inter IKEA Group 2019, p. 38). Inditex requires suppliers in breach of labor compliance to undergo a corrective action plan, and under some circumstances, Inditex may cease its business relationships with the supplier (Inditex 2020a). Cho et al. (2019) study the same two remedial actions, rectifying and dropping a supplier, with a focus on child labor violations. Porteous et al. (2015) empirically identify contract termination (i.e., dropping) and supplier training (a practice often employed to rectify a supplier) as two of the most effective instruments to reduce supplier violations. The fact that procurement contracts are commonly incomplete on social responsibility (Letizia and Hendrikse 2016) also leaves room for different responses to similar violations. In accordance with these practices, in our model we consider the actions of *rectifying* or *dropping* a supplier once the buyer detects a violation.

Many large buyers have enough leverage to act on a supplier in violation, even one in tier 2. For example, a buyer may provide a tier 1 supplier with a list of authorized or denied tier 2 suppliers. Removing a firm from the authorized list, or adding it to the denied list, effectively drops it from the supply network. Apple, Dell, Hewlett-Packard, Honda, IBM, LG Electronics, and Toyota all issue approved vendor lists for use by their tier 1 suppliers (Choi and Linton 2011). Inditex rules out suppliers that repeatedly fail social audits (Inditex 2020c) and forbids assigning work to third parties without Inditex's authorization (Inditex 2020b). Target's policy states that it may terminate its business relationship with any upstream supplier found noncompliant by an audit (Target Brands, Inc. 2020). In keeping with these examples, in our model, the buyer has the latitude to audit and, if necessary, drop not only direct suppliers but also indirect suppliers.

Even a wealthy buyer does not have unlimited resources to exhaustively audit the entire network (Chen et al. 2020a), thus facing the challenge of prioritizing suppliers to audit. Whereas some companies publicly state that they have a mechanism for prioritizing suppliers (e.g., Target Brands, Inc. 2020), many companies' reports and disclosures are silent on prioritization. Regardless, buyers would benefit from making such prioritization decisions deliberately and carefully. For example, auditing a supplier in tier 1 and auditing a supplier in tier 2 carry drastically different consequences if the supplier is found to be in violation and thus dropped—dropping a supplier in tier 1 removes all its *exclusive tier* 2 *suppliers* from the network. An exclusive supplier to a tier 1 firm supplies this firm only but no other tier 1 firm in the network. To capture the importance of prioritization, in our model the outcome of any previous audit alters the supply network and affects subsequent decisions.

We build a dynamic model to study the auditing of suppliers in a three-tier supply network for social responsibility compliance. The buyer starts with a known existing supply network with two tier 1 suppliers (see Figure 1 for an illustrative example). Each tier 1 supplier has its own base of tier 2 suppliers. Some of these tier 2 suppliers are exclusive to a tier 1 firm, whereas others are shared by the two tier 1 firms. Suppliers in the same tier manufacture a *perfectly substitutable* product. Each stage of the *auditing phase* consists of the following: the buyer selects a supplier to audit; the audit reveals whether the supplier is compliant; and in the event of noncompliance, the buyer decides to *rectify* the supplier or to *drop* it.

Figure 1. Example of Supply Network



Notes. An illustrative supply network consisting of the buyer c, tier-1, suppliers A and B, and tier-2 suppliers 1, 2, 3, and 4.

Dropping a supplier has the effect of also dropping any other firms relying solely on that dropped supplier (e.g., in Figure 1, if the buyer drops firm B, supplier 4 is also dropped). Once the buyer decides to conclude the auditing phase, the remaining supply network is carried to the *production phase*, where every firm competes with its peers to collectively determine the equilibrium quantities and prices. The equilibrium determines the buyer's profit from production activity.

Equipped with the model, we investigate the following interdependent decisions: (1) Which supplier should be audited? Should the buyer prioritize suppliers in an upper tier or a lower tier? More central or more peripheral? For example, in Figure 1, instead of auditing suppliers 1 and 2 individually, auditing firm A and dropping it if it fails may be a less expensive approach to avoid potential penalties arising from suppliers 1 and 2. However, doing so carves away a profitable part of the supply network. (2) When is it optimal to *drop* a noncompliant supplier, along with its dependents? When is it optimal to *rectify* the supplier? The buyer forfeits profit from production activity attributable to the suppliers dropped: the fewer the suppliers in the upstream markets, the less competitive the markets will be, yielding a higher input cost for the buyer. (3) When is it optimal to cease auditing and go straight to production? In other words, when is it in the best interest of the buyer to "see no evil, hear no evil"? By leaving a supplier unaudited, the buyer faces a penalty if a nongovernmental organization (NGO), law enforcement, or the media expose a violation at the supplier. Yet the buyer may prefer not to learn of noncompliance among some suppliers rather than be obligated to address any revealed problem.

Starting with questions (2) and (3), we show that the optimal auditing policy has two subphases. In the first subphase, the buyer audits a number of suppliers, dropping those in violation. In the second subphase, the buyer either audits all remaining suppliers, rectifying any in violation, or proceeds to production without any auditing. One implication of this two-subphase policy is that the buyer alters the shape of the network only during the first subphase, as it drops noncompliant suppliers. The buyer then retains the remainder of the network, either rectifying all noncompliant suppliers or turning a blind eye to any noncompliance that may still exist. The buyer would choose the latter only if the potential penalty from the exposure of a violation is lower than the costs of auditing and rectifying any remaining unaudited supplier.

Knowing that the buyer will drop any noncompliant suppliers in the initial phase of auditing, the choice of which supplier to audit (question (1)) involves many intricate trade-offs. First, if we were to focus only on tier 2 suppliers, the buyer would drop a *least valuable unaudited supplier* (LVUS). We show that the LVUS is any exclusive tier 2 supplier on the more crowded side of the network—the side with more tier 2 suppliers (e.g., suppliers 1 or 2 in Figure 1). The competition on that side of the network is already more intense (as a result of the larger number of tier 2 suppliers), so removing a tier 2 supplier from that side of the network does not raise the buyer's input cost as much. As the buyer trims the more crowded side, the network would progress toward a more balanced shape. Second, if we limit the buyer to auditing at most one supplier—thus, forcing the buyer to choose in which tier to audit—we find that the buyer will prioritize tier 1 over tier 2 when the violation penalty is high. By choosing to audit a tier 1 supplier (e.g., firm A in Figure 1), and dropping it if noncompliant, the buyer is able to drop all of the tier 1 firm's exclusive suppliers as well (suppliers 1 and 2), thus avoiding multiple potential violations at the cost of a single audit. Third, when the buyer is free to audit any number of suppliers in any tier, the buyer sometimes audits a supplier in a pivotal position in the network—say, a shared supplier in tier 2 or a supplier in tier 1—as a litmus test to gauge the viability of the network. The buyer makes subsequent auditing decisions based on the outcome of such tests, sometimes pursuing a "rescue operation" (following a path primarily auditing the next LVUS) to extract maximum value from the remaining network, sometimes embarking on a "kill mission" (auditing and dropping exhaustively starting from tier 1 to eradicate noncompliance).

2. Literature Review

The literature on socially responsible supply chain management covers various aspects, including supplier selection (Guo et al. 2016, Agrawal and Lee 2019), the effectiveness of incentives and penalties (Porteous et al. 2015), investment in supplier social responsibility capacity (Kraft et al. 2020), disclosure of the supplier list (Chen et al. 2019, Kalkanci and Plambeck 2020), unauthorized subcontracting (Caro et al. 2021), network-wide effects of violation penalty (Zhang et al. 2020), and consumer motives (Kraft et al. 2018).

Within this literature, our work is most closely related to the stream on auditing practices. See Dawande and Qi (2021) for a survey. One stream in the area considers a single buyer sourcing from a single supplier. Plambeck and Taylor (2016) show that under certain conditions, increasing auditing effort on the supplier motivates the supplier to hide misconduct. Through a comparison of alternative contracting arrangements, Chen and Lee (2017) establish that process auditing effectively reduces the risk of noncompliance by a supplier. Cho et al. (2019) study how a buyer combines auditing and pricing strategies to combat child labor. Chen et al. (2020b) study the collusion between the supplier and the auditor.

Recently, researchers have been considering the relationship between multiple firms in a supply network. When we consider multiple firms, we observe three approaches of auditing arrangements in practice and in the literature: (1) direct auditing by the buyer (or a third-party auditor representing the buyer), (2) delegating audits to other suppliers in the network, and (3) a coalition of buyers jointly conducting audits.

A first stream of literature compares direct auditing (approach (1)) and delegation (approach (2)). Huang et al. (2020) and Feng et al. (2020) study when the buyer should directly implement social responsibility standards at upstream suppliers versus delegating the task to midtier suppliers. In particular, Feng et al. (2020) consider supply networks with fixed material flow structure and find that the buyer gains more by engaging directly with upstream suppliers when the network has a complex structure.

A second stream compares direct auditing (approach (1)) and joint auditing (approach (3)). Focusing on a single supplier, Caro et al. (2018) and Fang and Cho (2020) compare the mechanism of independent, joint, and shared auditing by multiple buyers. Also on the subject of information sharing, Ha et al. (2018) study how sharing audit information between competitors interacts with sourcing decisions. Chen et al. (2020a) study the auditing behavior of two buyers situated in a W-shaped supply network (i.e., each buyer has one exclusive supplier and the two buyers share a third supplier). Their study shows that without coordination, each buyer chooses to audit its exclusive supplier, leading to a suboptimal combined profit for the buyers; with joint auditing, the buyers choose the shared supplier and avoid the inefficiency.

Different from both above-mentioned streams, we study approach (1) in a setting where a single buyer directly audits suppliers in a three-tier network. The buyer may decide to remove noncompliant suppliers from the network, allowing the topology of the network to evolve dynamically throughout the auditing process.

To understand the dynamic auditing decision, we also need to model how the topology of a network affects the competition that determines quantities and prices. This aspect of our model connects our work to the diverse literature on supply networks—for example, Acemoglu et al. (2012), Ang et al. (2017), Belavina (2017), Bimpikis et al. (2018), and Korpeoglu et al. (2020). The research closest to ours adopts a model of Cournot competition that endogenously determines equilibrium prices and quantities. The foundational work of Corbett and Karmarkar (2001), along with others, including Adida and DeMiguel (2011) and Bimpikis et al. (2019), studies supply networks with a complete market between adjacent tiers: every buyer in a downstream tier procures from every supplier in an upstream tier. Although we limit our model to two suppliers in the midstream tier, our work contributes to the literature by allowing arbitrary linkages between firms in adjacent tiers. Adida et al. (2016) also consider arbitrary linkages in a variant of the Cournot setup but with a different sequence of decisions. In an extension (Section 6.1), we show that the setup of Adida et al. (2016) preserves our main results. Our model allows us to discover how exclusive suppliers in different parts of the network contributes differently.

3. Model Description

The model consists of two phases, an auditing phase followed by a production phase, each consisting of multiple stages. We consider a single buyer auditing its existing supply network with two tiers of suppliers. Each supplier in the network can be compliant or noncompliant, which the buyer may discover with an audit. In each stage of auditing, the buyer chooses which supplier to audit, if any. If the audit finds noncompliance by the supplier, then the buyer either rectifies the supplier so that it becomes compliant or drops it from the supply network. Dropping a supplier may involve dropping dependent suppliers, as discussed in Section 3.1. The buyer may terminate the auditing phase at any point and carry the remaining network to the production phase, in which each firm chooses its supply quantities in competition with one another.

We consider our two-phase model (auditing followed by production) as a single auditing "cycle."[1] One could envision this auditing cycle to be a single iteration (e.g., auditing activities in a given year) of an ongoing multicycle auditing campaign (e.g., a multiyear program that continues for the foreseeable future), as motivated by the companies cited in the introduction. A more elaborate model would capture not only the "inner" dynamics of deciding the sequence of suppliers to audit (which we capture in our model) but also the "outer" dynamics of how the current cycle of auditing fits within the multicycle campaign. For example, such a model would need to track how a previously compliant supplier may fall into noncompliance over time, which then requires the periodic updating of the probability of noncompliance of each supplier. Any drop-or-rectify decision would need to account for a multitude of paths in future cycles with the associated economic effects.

Although compelling, we deemed such a model intractable. Therefore, we have chosen to bypass the outer dynamics of multiple cycles in favor of the inner dynamics within one cycle. We acknowledge that this is an approximation of an ongoing auditing effort. By focusing on auditing decisions in a single cycle, we are able to distill how the sequence of audits interacts with the supplier location in the network. The location affects the economic value of the supplier and the path of audits even within a single cycle.

We next describe in detail the supply network model, the auditing phase, and the production phase. We list the notation in Online Appendix A.

3.1. Supply Network

We model a three-tier supply network with a single buyer in tier 0, two suppliers in tier 1, and any number of suppliers in tier 2. Suppliers in the same tier manufacture a *perfectly substitutable* product.

We denote the *buyer* as C and the two *tier 1 firms* as A and B. Denote the set of tier 1 firms as $S(1) = \{A, B\}$. Let $S(2)$ denote the set of tier 2 firms, which we partition into three subsets: S_A, S_B, S_{AB}. The subset S_A is the set of *exclusive suppliers to firm* A, each of which sells to A but not to B. Similarly, the subset S_B is the set of *exclusive suppliers to firm* B. The subset S_{AB} is the set of *shared suppliers*, each selling to both firms A and B. We represent the *supply network* by the tuple $g = (S(1), S_A, S_B, S_{AB})$. We denote by $S_g = S(1) \cup S(2)$ the set of all suppliers in g. Throughout the auditing phase, the buyer may drop tier 1 firms, resulting in $S(1)$ having fewer than two firms. In particular, when $S(1) = \emptyset$ (or $S(2) = \emptyset$), we denote the resulting *null supply network* $g_\emptyset$. We denote by G the set of all supply networks.

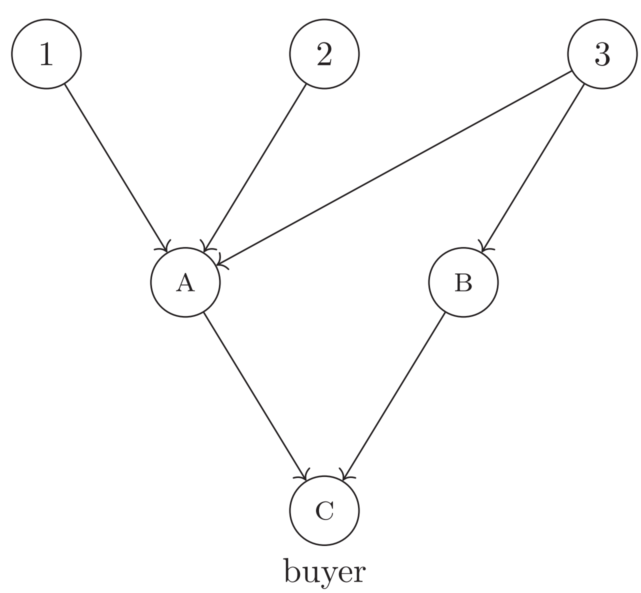
Given supply network $g \in G$, we denote by $t_A = |S_A|$ the number of exclusive suppliers to firm A, $t_B = |S_B|$ the number of exclusive suppliers to firm B, and $t_{AB} = |S_{AB}|$ the number of shared suppliers. We call the tier 1 firm with more tier 2 suppliers the *majority tier 1 firm* and the other the *minority tier 1 firm*. We call an exclusive supplier to the majority tier 1 firm a *majority-exclusive supplier* and an exclusive supplier to the minority tier 1 firm a *minority-exclusive supplier*.

The model uses the concept of *dependent suppliers*, as illustrated in Figure 2. Given a supplier i in network g, denote $D_g(i)$ as the set of *dependents* of i in g, each solely relying on i to sell to the buyer. Specifically, a supplier is always a dependent of itself. If i is a tier 1 firm, its dependents also include all its exclusive suppliers (i.e., if $i \in S(1)$, then $D_g(i) = S_i \cup \{i\}$). For example, in Figure 2, $D_g(A) = \{1, 2, A\}$. If i is a tier 2 supplier, then its dependents also include any tier 1 firm whose sole supplier is firm i (i.e., if $i \in S(2)$, then $D_g(i)$ includes a tier 1 firm $j \in \{A, B\}$ if and only if $S_j \cup S_{AB} = \{i\}$). In Figure 2, $D_g(3) = \{3, B\}$ and $D_g(1) = \{1\}$.

3.2. Auditing Phase

We now describe the auditing phase. Each supplier is either compliant (with probability $1 - u$) or noncompliant (with probability $u \in (0, 1)$). Through an audit, the buyer accurately discovers whether the supplier is compliant or noncompliant. (In Section 6.3 we consider an extension where the audits may be inaccurate.) Whether a supplier is compliant or not is independent across suppliers. In each stage of the auditing phase, the buyer decides whether to audit a supplier or to conclude the auditing phase and proceed to the production phase. If the buyer decides to audit (at cost $a \geqslant 0$), it selects an unaudited supplier. If the audit of that supplier reveals noncompliance, the buyer decides either to drop the noncompliant supplier from the supply network or to rectify it. When a supplier is dropped, its dependents are dropped as a consequence. For example, in Figure 2, dropping firm A would result in firms 1 and 2 being dropped, too. The buyer need not be concerned with noncompliance at those dropped dependent suppliers. When a supplier is rectified, the buyer incurs a rectification cost $r \geqslant 0$. We assume that a supplier completing rectification becomes compliant. The rectification cost r could involve training and education (Locke et al. 2009), identifying root causes and

Figure 2. Example of Supplier Dependence



Notes. Supply network $g = (\{A, B\}, \{1, 2\}, \emptyset, \{3\})$. Here, suppliers 1 and 2 are dependents of firm A, but A is not a dependent of 1 ot 2; firm B is a dependent of supplier 3, but 3 is not a dependent of B.

developing compliance solutions (Locke et al. 2009, Short et al. 2020), technical assistance (Locke et al. 2009), follow-up reaudits (Short et al. 2020), and subsidizing the purchase of equipment. In practice, the rectification cost may also depend on the size of the supplier and the nature of the violation, or it could manifest as an increase in the supplier's production cost. A more elaborate model could accommodate such intricacies of rectification.

We model the auditing phase as a Markov decision process for the buyer. A *state* consists of a supply network (tier 1 suppliers, their exclusive suppliers, and their shared suppliers) and the auditing status of each supplier (*unaudited* or *vetted*). An *unaudited* supplier becomes *vetted* if it passes an audit or undergoes successful rectification upon failing an audit; in either case, the buyer knows the supplier is compliant.[2] Specifically, a state is a tuple $\gamma = (g_\gamma, U_\gamma)$, where $g_\gamma = (S(1), S_{\text{A}}, S_{\text{B}}, S_{\text{AB}})$ is a supply network and $U_\gamma \subseteq S_{g_\gamma}$ is the set of suppliers that are currently *unaudited*. We omit the subscript γ whenever doing so causes no confusion. Any supplier $i \in S_g \backslash U$ is *vetted*. The *state space* is $\Gamma = \{(g, U) : g \in G, U \subseteq S_g\}$. The terminal states Γ_T are the supply networks with no more unaudited suppliers, $\Gamma_T = \{\gamma = (g, U) \in \Gamma : U = \emptyset\}$. In the terminal states, the auditing phase necessarily concludes, and the production phase begins. However, the buyer may choose to enter the production phase prior to reaching a terminal state (i.e., to proceed to production with unaudited suppliers).

To facilitate the formulation of the dynamic program, we define two operators that will be used when updating the state. Let $Z = \{(\gamma, i) : \gamma \in \Gamma, i \in U_\gamma\}$ be the set of pairs of a state and an unaudited supplier (in that state). The first mapping $\oplus : Z \to \Gamma$ changes a supplier from an unaudited to a vetted status (i.e., given state γ and unaudited supplier i in γ, $\gamma \oplus i$ is the state otherwise identical to γ but with a vetted i).[3] The operator $\oplus$ will be used when a supplier passes an audit or is rectified upon failing an audit. The second mapping $\ominus : Z \to \Gamma$ removes a supplier along with its dependents from a state; that is, given state γ and unaudited supplier i in γ, $\gamma \ominus i$ is the state otherwise identical to γ but with i and all its dependents removed.[4] The operator $\ominus$ is used when a supplier has failed an audit and is dropped from the network.

The buyer's set of *admissible actions* at state $\gamma \in \Gamma$ is $X_\gamma = \{\text{PP}\} \cup (\bigcup_{i \in U_\gamma} \{\text{AR}(i), \text{AD}(i)\})$. The action PP represents concluding the auditing phase and *proceeding* to the *production* phase. The action AR(i) represents *auditing* supplier i and *rectifying* i if the audit uncovers noncompliance. Following AR(i), regardless of whether the supplier passes the audit, the state transits from γ to $\gamma \oplus i$. Similarly, the action AD(i) represents *auditing* supplier i and *dropping* i (and its dependents) if the audit uncovers noncompliance. Following AD(i), the state transits from γ to $\gamma \oplus i$ if i is compliant (passes the audit) and to $\gamma \ominus i$ if i is noncompliant (fails the audit). In the notation for AD and AR, besides using the specific index for a supplier (e.g., i), we also use $\mathbb{e}_{\text{A}}$ to represent a generic unaudited exclusive supplier to firm A, $\mathbb{e}_{\text{B}}$ an exclusive supplier to firm B, and $\mathbb{s}$ a shared supplier (e.g., AD($\mathbb{e}_{\text{A}}$) represents the decision to audit and drop (if noncompliant) an exclusive supplier to firm A). Let $\bar{U} = \bigcup_{\gamma \in \Gamma} U_\gamma$ and $\bar{X} = \{\text{PP}\} \cup (\bigcup_{i \in \bar{U}} \{\text{AR}(i), \text{AD}(i)\})$. An *auditing policy* is a mapping $\xi : \Gamma \to \bar{X}$ such that $\xi(\gamma) \in X_\gamma, \forall \gamma \in \Gamma$. Let Ξ be the set of all auditing policies.

Given $\gamma = (g, U) \in \Gamma$, let $\pi(\gamma)$ be the buyer's *production profit*, which is a result of the equilibrium production activity on supply network g in the production phase (we uniquely determine this equilibrium in Section 4.1). We use $\nabla(\gamma, i) = \pi(\gamma) - \pi(\gamma \ominus i)$ to denote the loss in production profit as a result of the removal of a supplier $i \in U_\gamma$ (in state γ). We call $\nabla(\gamma, i)$ the *production value* of supplier i in state γ. A violation by a noncompliant supplier will be exposed in the production phase with probability $w \in (0, 1]$; for example, this revelation may arise from an investigation led by an NGO or a regulatory body. The possible exposure of violation is independent across suppliers. The buyer incurs a cost of $z \geqslant 0$ upon the exposure of a violation at each noncompliant supplier. Let $\zeta(\gamma)$ be the expected total penalty from violations on state γ. (If any penalty arises at all, it does so during the production phase.) By the independence of noncompliance and exposure of violations across suppliers, $\zeta(\gamma) = |U_\gamma| uwz$, because U_γ is the set of unaudited suppliers, each of which is noncompliant with probability u and costs the buyer a penalty z if exposed with probability w. The homogeneity of the penalty z across all suppliers implies that social responsibility violations are generic in the minds of the public, so the responses they elicit from the public are equally damning to the buyer. Therefore, we do not consider different types of violations that may differ in their severity (e.g., slave labor versus spillage of animal effluent) and, thus, in the penalties they impose on the buyer. In Section 6.2 we relax the homogeneity assumption by allowing the penalty to depend on the supplier's tier.

We define $V^* : \Gamma \to \mathbb{R}$ as the optimal value function. Let $\widetilde{V}^*(\gamma, x)$ be the expected value of choosing action $x \in X_\gamma$ when in state $\gamma \in \Gamma$ and following an optimal policy thereon. Then

$$V^*(\gamma) = \max_{x \in X_\gamma} \widetilde{V}^*(\gamma, x), \quad \forall \gamma \in \Gamma, \tag{1}$$

where

$$\widetilde{V}^*(\gamma, \text{PP}) = \pi(\gamma) - \zeta(\gamma), \tag{2}$$

and for $i \in U_\gamma$,

$$\widetilde{V}^*(\gamma, \text{AD}(i)) = -a + (1-u)V^*(\gamma \oplus i) + uV^*(\gamma \ominus i), \quad (3)$$

$$\widetilde{V}^*(\gamma, \text{AR}(i)) = -a + (1-u)V^*(\gamma \oplus i) + u(-r + V^*(\gamma \oplus i)), \quad (4)$$

$$= -a - ur + V^*(\gamma \oplus i). \quad (5)$$

Equation (2) reflects the buyer's value when it takes action PP, which consists of the production profit minus the expected penalty of violation. Equations (3) and (4) are the buyer's values when it takes actions $\text{AD}(i)$ and $\text{AR}(i)$, respectively. The values consist of an auditing cost a and the weighted average of values in consequent states when the supplier passes (with probability $1-u$) or fails (with probability u) the audit. In a terminal state $\gamma \in \Gamma_T$, $X_\gamma = \{\text{PP}\}$, so $V^*(\gamma) = \pi(\gamma) - \zeta(\gamma)$. An optimal auditing policy $\xi \in \Xi$ solves $\xi(\gamma) \in \arg\max_{x \in X_\gamma} \widetilde{V}^*(\gamma, x)$ for any $\gamma \in \Gamma$.

Throughout the auditing phase, the topology of the supply network and the status of each remaining supplier evolve with the progression of auditing. Once the auditing phase concludes (either the buyer decides to proceed to production or the Markov decision process enters a terminal state), the remaining supply network is carried to the production phase, where the buyer's production profit is determined.

Motivated by powerful global buyers such as Apple and IKEA, we assume that the buyer in our model is powerful enough to remove a tier 2 supplier with cause (i.e., failing an audit). However, we do not allow the buyer to insist that a tier 1 supplier drop a tier 2 supplier *without cause*. Rather, the buyer typically influences tier 1 firms' choice of tier 2 suppliers through lists of disallowed tier 2 suppliers or approved vendor lists from which tier 2 suppliers may be chosen. Without sufficient cause, it would be odd for the buyer to deny an existing tier 2 supplier.[5]

3.3. Production Phase

In the production phase, each firm in the network chooses its production quantity to maximize its profit, given the unit input price and anticipating the downstream demand. The upstream firms' chosen quantities determine a downstream firm's input price, as we later describe. All firms in S_A and S_AB compete to supply firm A, all firms in S_B and S_AB compete to supply firm B, and firms A and B compete to supply the buyer. This is similar to Corbett and Karmarkar (2001), other than we limit ourselves to three tiers and two tier 1 firms but allow more general relationships between firms in adjacent tiers. Specifically, in Corbett and Karmarkar (2001), all firms in an upstream tier are shared suppliers of all downstream firms; using our notation, they have $S(2) = S_\text{AB}$ and $S_\text{A} = S_\text{B} = \emptyset$.

Corresponding to the three tiers in the supply network, there are three stages in the production phase. In the first stage, the firms in S_A and S_AB choose the quantities they will supply to firm A, which establishes $p_{(2)\text{A}}$, the selling price of those tier 2 suppliers to firm A (whenever a number appears in parentheses in a subscript, it refers to the tier in the network). In parallel, the firms in S_B and S_AB choose their quantities to establish $p_{(2)\text{B}}$, their selling price to firm B. In particular, a shared supplier in tier 2 may sell to firms A and B at different prices. In the second stage, firms A and B choose quantities, which results in $p_{(1)}$, the selling price of the tier 1 firms to the buyer. In the third stage, the buyer chooses a quantity to establish $p_{(0)}$, the selling price of the buyer to downstream customers. Let q_j be the total quantity produced by firm j. The unit production cost of a tier k firm is v_k; let $v_T = \sum_{k=0}^{2} v_k$, which is the total production cost embedded in each unit of the final product. Next we describe these three stages in detail, in reverse order.

3.3.1. Third Stage: Buyer's Problem.

In the third stage of the production phase, the buyer C faces an exogenous linear aggregate demand from downstream customers characterized by the following inverse demand function:

$$p_{(0)}(q_\text{C}) = \alpha - \beta q_\text{C}, \quad (6)$$

where $\beta > 0$. Assume $\alpha \geqslant v_T$ to ensure the supply chain is profitable. The buyer takes the price of the input $p_{(1)}$ as given and chooses production quantity q_C in decision space $C_\text{C} = \mathbb{R}$ to maximize profit:

$$(P_0) \qquad \max_{q_\text{C} \in C_\text{C}} \{\pi_\text{C}(q_\text{C}) \equiv (p_{(0)}(q_\text{C}) - v_0 - p_{(1)})q_\text{C}\}. \quad (7)$$

By Proposition C.1 in Online Appendix C.1, given the buyer's input price $p_{(1)}$, there exists a unique optimal quantity q_C^*, which solves the buyer's problem P_0. We denote the resulting inverse demand function faced by the tier 1 firms by $p_{(1)}^*(q_\text{C})$.

3.3.2. Second Stage: Tier 1 Firms' Game.

In the second stage of the production phase, tier 1 firm $i \in S(1)$, anticipating inverse demand $p_{(1)}^*(q_\text{C})$ and taking the input prices $p_{(2)i}$ as given, chooses production quantity q_i in strategy space $C_i = \mathbb{R}$ to maximize its profit:

$$\pi_i = \left(p_{(1)}^*(q_\text{C}) - v_1 - p_{(2)i}\right)q_i \quad (8)$$

subject to the market clearing condition

$$q_\text{C} = \sum_{i' \in S(1)} q_{i'}. \quad (9)$$

In other words, when there are two tier 1 firms, they engage in Cournot competition for the buyer's business. Denote a strategy profile of the tier 1 firms

$\mathbf{q}_{(1)} = (q_i)_{i\in S(1)} \in \prod_{i\in S(1)} C_i$ (i.e., a vector of tier 1 supply quantities). We substitute (9) into (8) to write tier 1 firm i's payoff function as

$$\pi_i\left(\mathbf{q}_{(1)}\right) = \left(p^*_{(1)}\left(\sum_{i'\in S(1)} q_{i'}\right) - v_1 - p_{(2)i}\right)q_i. \tag{10}$$

Let the game in the second stage be the strategic form game $P_1 = (S(1), (C_i)_{i\in S(1)}, (\pi_i)_{i\in S(1)})$. By Proposition C.2 in Online Appendix C.1, given the tier 1 vector of input prices $\mathbf{p}_{(2)} = (p_{(2)i})_{i\in S(1)}$, there exists a unique equilibrium in pure strategies $\mathbf{q}^*_{(1)}$ of the game P_1. We denote the resulting inverse demand function faced by the tier 2 firms supplying tier 1 firm i by $p^*_{(2)i}(\mathbf{q}_{(1)})$.

3.3.3. First Stage: Tier 2 Suppliers' Game. In the first stage of the production phase, each tier 2 supplier j chooses $s_{j,i}$, the quantity it produces for its tier 1 customer firm i. A tier 2 supplier j anticipates inverse demand $p^*_{(2)i}(\mathbf{q}_{(1)})$ for $i \in S(1)$. An exclusive supplier j to tier 1 firm $i \in S(1)$ chooses $\mathbf{s}_j = s_{j,i} \geqslant 0$ to maximize its profit,

$$\pi_j = \left(p^*_{(2)i}\left(\mathbf{q}_{(1)}\right) - v_2\right)s_{j,i}. \tag{11}$$

A shared supplier j chooses the vector of supply quantities $\mathbf{s}_j = (s_{j,i})_{i\in S(1)} \geqslant 0$ to maximize its profit:

$$\pi_j = \sum_{i\in S(1)} \left(p^*_{(2)i}\left(\mathbf{q}_{(1)}\right) - v_2\right)s_{j,i}. \tag{12}$$

The tier 2 suppliers' decisions are subject to the market-clearing condition:

$$q_i = \sum_{j'\in S_i\cup S_{\text{AB}}} s_{j',i}, \quad \forall i \in S(1). \tag{13}$$

In other words, for each firm i in tier 1, the tier 2 suppliers in S_i and S_{AB} engage in Cournot competition to supply firm i. When there are two tier 1 firms, a shared tier 2 supplier competes simultaneously for the business of each tier 1 firm.

To make the strategy space of a tier 2 supplier compact for proving existence and uniqueness of the equilibrium, we assume there exists (arbitrarily large) theoretical maximum capacity $M > 0$ such that $s_{j,i} \leqslant M$ for any tier 2 supplier $j \in S(2)$ and its tier 1 customer i. That is, the strategy space of supplier $j \in S(2)$ is $C_j = \{\mathbf{s}_j : 0 \leqslant s_{j,i} \leqslant M\}$. Denote a strategy profile of tier 2 suppliers as $\mathbf{s}_{(2)} = (\mathbf{s}_j)_{j\in S(2)} \in \prod_{j\in S(2)} C_j$. We substitute (13) into (11) and (12) to obtain the payoff functions $\pi_j(\mathbf{s}_j)$ of the exclusive and shared suppliers, respectively. Let the game in the first stage be the strategic form game $P_2 = (S(2), (C_j)_{j\in S(2)}, (\pi_j)_{j\in S(2)})$. We denote an equilibrium in pure strategies as $\mathbf{s}_{(2)}{}^* = (\mathbf{s}^*_j)_{j\in S(2)} = (((s^*_{j,i})_{j\in S_i})_{i\in S(1)}, ((s^*_{j,i})_{i\in S(1)})_{j\in S_{\text{AB}}})$. Let $\mathbf{p}^*_{(2)} = (p^*_{(2)i})_{i\in S(1)}$. Let $q^*_j = s^*_{j,i}$ if $j \in S_i$, $i \in S(1)$, and let $q^*_j = \sum_{i\in S(1)} s^*_{j,i}$ if $j \in S_{\text{AB}}$.

3.3.4. Production Phase Equilibrium. Let $\mathbf{s}^*_{(2)}$ be an equilibrium of the game P_2 in the first stage and $\mathbf{p}^*_{(2)}$ the resulting selling prices of the tier 2 suppliers. Given $\mathbf{p}^*_{(2)}$, let $\mathbf{q}^*_{(1)}$ be an equilibrium of the game P_1 in the second stage and $p^*_{(1)}$ the resulting selling price of the tier 1 firms. Given $p^*_{(1)}$, let q^*_{C} be an optimal solution to the buyer's problem P_0 in the third stage and $p^*_{(0)}$ the resulting selling price of the buyer. We call the tuple of prices and quantities $(p^*_{(0)}, p^*_{(1)}, \mathbf{p}^*_{(2)}, q^*_{\text{C}}, \mathbf{q}^*_{(1)}, \mathbf{s}^*_{(2)})$ a *production phase equilibrium*. In the next section we show that this equilibrium is unique, and we fully characterize the equilibrium.

The role of the production model in our paper is to establish the economic values of suppliers to the buyer. To that end, the production model we previously described applies Cournot competition to our three-tier network. In Section 6.1, we study an alternative production model that follows the competition model in Adida et al. (2016) in which the buyer makes its quantity decision first, followed by the tier 1 firms, and finally the tier 2 firms. We show that this alternative model preserves our results. Importantly, as we discuss in Section 6.1, our results rely only on two intuitive properties that the buyer's profit must satisfy in the production model.

4. Production Phase Results

In the spirit of backward induction, we present the results for the two phases in reverse order: the results of the production phase in this section are followed by the results of the auditing phase in Section 5.

4.1. Production Phase Equilibrium

We are now ready to present the existence and uniqueness of the production phase equilibrium. We relegate all proofs to the online appendices.

Theorem 1. *There exists a unique production phase equilibrium* $(p^*_{(0)}, p^*_{(1)}, \mathbf{p}^*_{(2)}, q^*_{\text{C}}, \mathbf{q}^*_{(1)}, \mathbf{s}^*_{(2)})$ *in every supply network* $g \in G\backslash\{g_\emptyset\}$.

Let π^*_{C} be the buyer's profit in the unique equilibrium in Theorem 1, obtained by substituting the equilibrium quantities and prices in (7). For any state in the auditing phase $\gamma = (g, U) \in \Gamma$, the buyer's *production profit* $\pi(\gamma)$ is given by π^*_{C} in the equilibrium that arises in network g.[6]

Recall that $t_{\text{A}} = |S_{\text{A}}|$, $t_{\text{B}} = |S_{\text{B}}|$, and $t_{\text{AB}} = |S_{\text{AB}}|$. Without loss of generality, we index the majority tier 1 firm as A (i.e., $t_{\text{A}} \geqslant t_{\text{B}}$). Proposition C.5 in Online Appendix C.2 solves for the equilibrium supply quantities of all firms, which allows us to express the prices and profits as well. The proposition delivers two key results: (1) Beyond market demand parameters (α and β) and the total production cost (v_T), the only factor that determines the production phase equilibrium is the topology of the supply network, captured by t_A, t_B, t_{AB}. (2) If the majority tier 1 firm A has so many exclusive tier 2 suppliers (relative to shared suppliers and firm B's exclusive suppliers) that $t_A \geqslant 2t_B + 2t_{AB} + 2$, firm A's upstream market will be too competitive for any shared supplier to profitably participate in it. Each shared supplier will behave as if it is an exclusive supplier to minority tier 1 firm B.

4.2. Comparative Statics

We seek to understand the value of the contribution by each supplier to the buyer's production profit, which guides the buyer's decisions in the auditing phase. Let m_C^*, m_A^*, and m_B^* be the margins of the buyer, firm A, and firm B in equilibrium, i.e., $m_C^* = p_{(0)}^* - p_{(1)}^* - v_0$, $m_A^* = p_{(1)}^* - p_{(2)A}^* - v_1$, and $m_B^* = p_{(1)}^* - p_{(2)B}^* - v_1$. Let $\rho_A^* = \frac{q_A^*}{q_C^*}$ and $\rho_B^* = \frac{q_B^*}{q_C^*}$ be the market shares of the two tier 1 firms in equilibrium. As in Section 4.1, we index the majority tier 1 firm as A (i.e., $t_A \geqslant t_B$).

Theorem 2. *Given a supply network $g \in G$, adding a tier 2 supplier changes the equilibrium values of the variables in Table 1 as follows:*

(a) *Adding a majority-exclusive supplier changes the equilibrium values as in column* (I).

(b) *Adding a minority-exclusive supplier changes the equilibrium values as in column* (II).

(c) *Adding a shared supplier changes the equilibrium values as in column* (III) *if $t_A \leqslant 2t_B + 2t_{AB} + 2$ and as in column* (II) *otherwise.*

Theorem 2(a) and 2(b) reflect the following intuitive observation: adding an exclusive supplier to tier 1 firm $i \in \{A, B\}$ gives firm i greater advantage in competition—increasing its quantity q_i^*, margin m_i^*, profit π_i^*, and market share ρ_i^* and decreasing the same metrics for the other tier 1 firm. Theorem 2(c) shows that $t_A > 2t_B + 2t_{AB} + 2$, firm A dominates the supply network so much that the shared suppliers behave as exclusive suppliers to firm B, in which case adding a shared supplier has the same effect as adding an exclusive supplier to firm B, as shown in column (II) of Table 1. The only ambiguity arises when $t_A \leqslant 2t_B + 2t_{AB} + 2$, and we add a shared supplier. In this case, as shown in column (III), whereas the effect on firm B is clear, the effect on firm A's quantity, margin, and profit is not. Consider firm B, which has less market power: adding a shared supplier boosts firm B's power more than it boosts that of firm A, which enjoyed greater power to begin with. Indeed, firm B's quantity, margin, profit, and market share all improve, at the expense of firm A's market share. The following result resolves the ambiguity of the effect on firm A's quantity, margin, and profit.

Table 1. Comparative Statics in the Production Phase

Variable	(I)	(II)	(III)
q_C^*, m_C^*, π_C^*	+	+	+
q_A^*, m_A^*, π_A^*	+	−	+/−
q_B^*, m_B^*, π_B^*	−	+	+
$p_{(1)}^*$	−	−	−
$p_{(2)A}^*$	−	−	−
$p_{(2)B}^*$	−	−	−
ρ_A^*	+	−	−
ρ_B^*	−	+	+

Notes. Shown is how the equilibrium value of each variable changes as the number of tier 2 suppliers increases by 1. (See Theorem 2 for details.) The plus sign (+) indicates the variable increases, the minus sign (−) indicates the variable decreases, and the plus/minus sign (+/−) indicates that there exist both instances of the variable increasing and decreasing depending on the specific topology of the supply network (captured by t_A, t_B, t_{AB}).

Proposition 1. *Given supply network $g \in G$, where $t_A \leqslant 2t_B + 2t_{AB} + 2$, there exists a threshold $\theta(t_B, t_{AB})$ such that adding a shared supplier to the network increases the majority firm A's quantity q_A^*, margin m_A^*, and profit π_A^* if and only if $t_A < \theta(t_B, t_{AB})$.*[7]

When t_A is relatively small, adding a shared supplier benefits both firm B (as we have seen in Theorem 2) and firm A. When t_A is relatively large, adding a shared supplier benefits the minority tier 1 firm B but hurts the majority tier 1 firm A. The reason is that although adding the shared supplier makes firm A's input market more competitive, directly benefiting firm A, the addition benefits the minority tier 1 firm B even more. In fact, the boost in firm B's power improves its position so much in the downstream competition with firm A that it rebounds to hurt firm A by overwhelmingly slashing its margin.

Theorem 2 shows that regardless of where we add the tier 2 supplier, the addition always increases the quantity, margin, and profit of the buyer. The next proposition ranks the buyer's gain from the addition of a supplier based on the supplier's location in the network.

Proposition 2. *Given supply network $g \in G$,*

(a) *adding a shared supplier to g induces a strictly greater increase in the buyer's quantity q_C^*, margin m_C^*, and profit π_C^* than adding a minority-exclusive supplier; and*

(b) *adding a minority-exclusive supplier to g induces a greater or equal increase in the buyer's quantity q_C^*, margin m_C^*, and profit π_C^* than adding a majority-exclusive supplier.*

For the buyer, a shared supplier is more valuable than a minority-exclusive supplier (Proposition 2(a)), which, in turn, is more valuable than a majority-exclusive supplier (Proposition 2(b)). Generally speaking, the greater the upstream competition, the better off the buyer. Adding a shared supplier intensifies the competition in both tier 1 firms' input markets, which then intensifies the competition in the buyer's input market more than adding an exclusive supplier would. Adding an exclusive supplier to the minority tier 1 firm helps to elevate the position of the minority tier 1 firm in the competition for the buyer's business, decreasing the buyer's input cost more than adding an exclusive supplier to the majority tier 1 firm would.

Because dropping a supplier dilutes competition in a way similar to a horizontal merger, we discuss our results in light of the literature on horizontal mergers in supply networks. Cho (2014) considers two effects of merger: a synergy effect (lower variable cost per unit because of, e.g., economies of scale) and the competition effect (less competition with fewer peers). Dropping a shared supplier in our model has the same effect as a merger in the model of Cho (2014) if there is no synergy effect (i.e., the variable cost per unit remains the same). Korpeoglu et al. (2020) consider a market game to study how the number of firms in each tier affects the equilibrium. The comparative statics in our model is generally consistent with Cho (2014) and Korpeoglu et al. (2020) in the corresponding cases. Importantly, as in Corbett and Karmarkar (2001), both Cho (2014) and Korpeoglu et al. (2020) consider only supply networks in which every supplier in an upstream tier sells to every buyer in a downstream tier. They do not consider exclusive suppliers or compare exclusive and shared suppliers as we do.

5. Auditing Phase Results

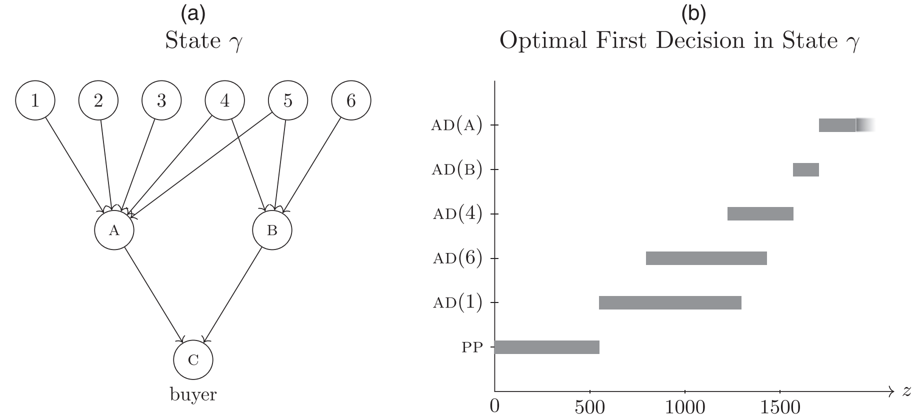
We begin with an example that illustrates some intriguing properties of the optimal auditing behavior, which we later explain in the following sections.

Example 1. We consider how the optimal first auditing decision varies with the value of penalty z. Consider the state shown in Figure 3(a) where all suppliers are unaudited. Let the parameters be $\alpha = 190, \beta = 1.4, v_T = 1.5, a = 75,\ r = 860, u = 0.47$, and $w = 0.46$. Figure 3(b) presents the optimal first decision as penalty z varies.

With *low* z ($z < 550$), the potential penalty is too low to justify any audit by the buyer; the buyer proceeds to the production phase directly. With *high* z ($z > 1570$), the penalty is so great that the buyer turns its attention to the tier 1 firms. It is optimal for the buyer to start with auditing a tier 1 supplier and dropping it if noncompliant (AD(B) for $1570 < z < 1704$ and AD(A) for $z > 1704$). Dropping a tier 1 firm enables the buyer to drop all dependent tier 2 suppliers, thus avoiding the expense of auditing them or any associated violation penalties. In short, the penalty is so high that the buyer is willing to remove an entire side of the supply network rather than risk carrying unaudited suppliers through to production.

With *intermediate* z ($550 < z < 1570$), the buyer focuses the AD effort on the tier 2 suppliers. As z increases from 550, the optimal first audit is AD(1) (or any majority-exclusive supplier). As z further increases, AD(6) (the minority-exclusive supplier) also becomes an optimal first audit. Similarly, AD(4) (or any shared supplier) becomes an optimal first audit as z increases further up to 1570. As seen in Figure 3(b), there are ranges within $550 < z < 1570$ in which multiple auditing decisions are optimal: for example, for values of z between 1223 and 1297, AD(1), AD(4), and AD(6) are all equally good auditing decisions. For values of z between 1297 and 1431, AD(4) (or AD(5)) and AD(6) are the only optimal first auditing decisions; for values of z between 1431 and 1570, AD(4) (or the other shared supplier) becomes the unique optimal auditing decision. It is peculiar that the buyer would

Figure 3. State γ and Optimal First Decision in γ in Example 1



Notes. In panel (a), every supplier is unaudited in γ. In panel (b), as penalty z increases, the optimal decision at γ shifts in order from PP, to AD(1) (or any other exclusive supplier to firm A), to AD(6), to AD(4) (or the other shared supplier 5), to AD(B), and eventually to AD(A).

optimally choose supplier 6, a minority-exclusive supplier, or supplier 4, a shared supplier, as the first firm to audit. After all, Proposition 2 identifies supplier 1 (or 2 or 3) as the least valuable supplier within tier 2 to the buyer. One may therefore intuit that among tier 2 suppliers, the buyer would prioritize auditing supplier 1 (or any other majority-exclusive supplier), which is less valuable to the buyer when it reaches the production phase but carries as much penalty as any other supplier in the event of a violation. Yet, as the example shows, the buyer may find it optimal to start with an audit of a minority-exclusive or a shared supplier with greater direct damage to the buyer's production profit when dropped. Why would the buyer start its audit with a minority-exclusive or shared supplier and risk losing more production profit in the event it is dropped? □

5.1. Two Subphases of Auditing

We identify a surprisingly simple property of an optimal policy. The buyer will first *audit* and *drop* some suppliers; we call this the AD *subphase*. Only after the buyer ceases the AD subphase will it then proceed to what we refer to as the RP *subphase*. In the RP subphase the buyer either audits and *rectifies* all remaining suppliers in an arbitrary sequence or proceeds to the *production* phase directly.

Theorem 3. *There exists an optimal policy $\xi^* \in \Xi$ with the property that auditing decisions are divided into two subphases:*

(a) AD *subphase: to audit and drop* (AD) *some suppliers* (*or none*), *followed by*

(b) RP *subphase: to audit and rectify* (AR) *all remaining unaudited suppliers in an arbitrary sequence if $a + ur \leqslant uwz$ or to proceed to production* (PP) *if $a + ur \geqslant uwz$.*

Under the optimal policy in Theorem 3, any auditing and dropping activity (AD subphase) will precede any rectification activity (in the RP subphase). Once the buyer starts rectifying suppliers, it will not return to dropping suppliers again. The separation of AD and AR activities into subphases is intuitive: the buyer would not want to rectify a supplier only to find out later that it becomes "collateral damage"—dropped as a dependent of another dropped supplier. The theorem is, however, silent on the length of the AD subphase and when the RP subphase will commence. Specifically, the commencement of the RP subphase depends on the outcome of the audits conducted in the AD subphase.

Once in the RP subphase, now that the AD activity is over, the topology of the supply network will not change hereafter. This sets in stone the production profit, determined solely by the supply network, thereby leaving the buyer with the following decision for each unaudited supplier: whether to audit and rectify it or to allow it to enter the production phase unaudited. In deciding this, the buyer compares the cost of auditing the supplier and rectifying it if it fails the audit ($a + ur$) against the penalty that arises if an unaudited supplier is exposed to be noncompliant (uwz). Because all unaudited suppliers are identical (other than their location in the network), this comparison is identical for all suppliers. Hence in the RP subphase, the buyer either proceeds to production directly or audits and rectifies *all* remaining suppliers. We denote the cost associated with each unaudited supplier in the RP subphase $c_{\text{RP}} \equiv (uwz) \wedge (a + ur)$. In the scenario that the buyer stops short of auditing all suppliers in the AD subphase and proceeds to the production phase directly in the RP subphase, the optimal policy manifests as "see no evil, hear no evil": the buyer conducts no further audits, willing to tolerate some level of noncompliance. We revisit this scenario in Section 5.2.

The optimal behavior in the RP subphase leads to the following result.

Corollary 1. *At state $\gamma \in \Gamma$, if the optimal policy ξ^* is already in the* RP *subphase,*

$$V^*(\gamma) = \pi(\gamma) - c_{\text{RP}}|U_\gamma|. \tag{14}$$

In the rest of the paper, we focus on policies that consist of the two subphases described in Theorem 3. In particular, in each stage of the auditing phase, we only need to consider the actions to audit and drop a supplier (AD) and the action to proceed to the RP subphase. We introduce a new action, RP, that is a shorthand for "audit and rectify (AR) all remaining unaudited suppliers if $a + ur < uwz$ and proceed to production (PP) otherwise."

In the next example, we explore how the shape of the supply network affects the extent of auditing in each subphase.

Example 2. We fix the total number of tier 2 suppliers to six. Table 2 shows the expected number of suppliers audited, broken down to those audited with the intention to drop (AD) and those audited with the intention to rectify (AR), throughout the auditing horizon. The expectation is taken over the outcome of each audit (pass or fail), which affects subsequent optimal auditing paths. Each cell corresponds to a different supply network shape, fixing the number of tier 2 suppliers to six. As one moves down each column, the number of shared suppliers (t_{AB}) increases. As one moves rightward along each row, the network becomes more lopsided, with firm A's side of the network becoming more crowded (i.e., $t_{\text{A}} - t_{\text{B}}$ increases). The parameters are such that $a + ur \leqslant uwz$ so that the buyer will audit and rectify all remaining suppliers (rather than directly proceeding to production) in the RP subphase.

Table 2. Example 2: Expected Number of Suppliers Audited for Various Shapes of the Initial Supply Network

	$t_A - t_B$						
	0	1	2	3	4	5	6
	$t_A = t_B$			balanced ⟷ lopsided			$t_A = 6, t_B = 0$
t_{AB} 0	$\overbrace{4.16}^{\text{AD}} + \overbrace{1.26}^{\text{AR}} = 5.42$		3.95 + 1.47 = 5.42		4.28 + 1.32 = 5.59		4.34 + 0.08 = 4.42
1		4.75 + 0.92 = 5.67		3.66 + 2.25 = 5.91		4.28 + 1.32 = 5.59	
2	4.53 + 1.76 = 6.29		3.94 + 2.22 = 6.16		3.66 + 2.25 = 5.91		
3		4.48 + 1.92 = 6.40		3.80 + 2.35 = 6.16			
4	4.94 + 1.71 = 6.65		4.37 + 2.03 = 6.40				
5		4.94 + 1.71 = 6.65					
6	4.13 + 3.87 = 8.00						

Notes. The shape of the initial supply network is represented by the number of shared suppliers t_{AB} (rows) and the difference in the numbers of majority-exclusive and minority-exclusive suppliers $t_A - t_B$ (columns) with six tier 2 suppliers in total ($t_A + t_B + t_{AB} = 6$). All suppliers are initially unaudited. Each cell presents the expected number of suppliers audited throughout the auditing phase given the different shape. The first number in each equation is the expected number of suppliers audited for dropping (AD); the second number is that of suppliers audited for rectification (AR). Their sum is the expected total number of suppliers audited. The parameters are $\alpha = 360$, $\beta = 3.8$, $v_T = 2.3$, $a = 400$, $r = 260$, $z = 4{,}000$, $u = 0.43$, and $w = 0.91$, such that the optimal action in the RP subphase is to audit and rectify.

There are eight suppliers in the network (except in the upper right corner cell, where the supply chain has only one tier 1 firm—hence seven suppliers in total), yet the expected number of audits conducted is fewer than eight in all but one cell. Given that the buyer will audit and drop some supplier and then audit and rectify all remaining suppliers, why is the total number of audits ever fewer than eight? This happens because when the buyer drops a supplier, its dependents are dropped as collateral damage, even though they never undergo an audit. The buyer can audit fewer than eight suppliers and still end up with a fully vetted network.

Neither the number of suppliers audited and dropped nor the number of suppliers audited and rectified exhibits any monotonicity as the supply network becomes more lopsided (e.g., along the first row) or as the number of shared suppliers increases (e.g., along the first column). The lack of monotonicity intimates opposing forces at play as the network topology changes. We will return to this example to explain the opposing forces.[8] □

5.2. Optimal Auditing Sequence in Tier 2: Toward a Balanced Supply Network

In this subsection we consider the sequence of suppliers the buyer will audit and drop in the AD subphase. As Example 1 shows, any supplier may emerge as the buyer's optimal choice for the first audit as a single parameter of the model is varied. Despite this fickle behavior, once we limit auditing to the tier 2 suppliers, we demonstrate that the optimal auditing sequence is determined by the value of each firm, which, in turn, depends on the firm's location in the network. Later in Section 5.3, we consider the scenario where all suppliers are unaudited to shed light on the question from Example 1.

Recall that we define $\nabla(\gamma, i)$ as the production value of supplier i for the buyer at state γ: $\nabla(\gamma, i) = \pi(\gamma) - \pi(\gamma \ominus i)$. We now introduce an intuitive property of the supply network—specifically, a condition on production profit π.[9]

Condition 1 (Decreasing Differences of Production Profit)**.** *For any $\gamma \in \Gamma \backslash \{\gamma_1\}$ in which every tier 1 firm is vetted (i.e., $S(1) \cap U_\gamma = \emptyset$), and for any $i, i' \in U_\gamma$,*

$$\nabla(\gamma, i') \leqslant \nabla(\gamma \ominus i, i'). \tag{15}$$

Condition 1 says that the production value of a supplier is greater in smaller supply networks. This is intuitive because each additional supplier adds to the buyer's profit (as a result of increased competition leading to lower input prices) but to a lesser extent than the previous one. Using the closed-form expressions in Proposition C.5 in the online appendix, we have algebraically verified Condition 1 for any network with up to 100 tier 2 suppliers.

We now define a concept that is then used in the subsequent result.

Definition 1. Let $\gamma \in \Gamma$ and $i \in U_\gamma$. If $\nabla(\gamma, i) \leqslant \nabla(\gamma, i')$ for every $i' \in U_\gamma$, then we call i a *least valuable unaudited supplier*, or LVUS, in γ.[10]

An LVUS is an unaudited supplier that carries the least value to the buyer's production profit. We now show that the LVUS is the next firm to audit when auditing tier 2 firms only.

Theorem 4. *Suppose every tier* 1 *firm is vetted. Under Condition* 1*, in any state $\gamma \neq \gamma_1$, it is optimal for the buyer to* *audit and drop (if noncompliant) an LVUS i if* $a + u\nabla(\gamma, i) < c_{\text{RP}}$ *and to proceed to the* RP *subphase if* $a + u\nabla(\gamma, i) \geqslant c_{\text{RP}}$.

By Theorem 4, the buyer should audit and drop an LVUS i, provided that the cost of auditing, a, plus the potential loss of production profit by dropping i, $u\nabla(\gamma, i)$, is less than the expected cost in the RP subphase, c_{RP}, and proceed to production otherwise. Theorem 4 cements the buyer's optimal policy into a simple rule: audit and drop an LVUS, a supplier that carries the least production value, until the stage at which even an LVUS carries more value than the expected costs associated with the supplier in the RP subphase (captured by c_{RP}). Once at that stage, the buyer should proceed to the RP subphase: either rectify all noncompliant suppliers identified by exhaustive auditing or just proceed to production and brace itself for any damages from the exposure of violation at unaudited suppliers. When even an LVUS carries sufficient production value to justify retention, and the penalty for violation is not high enough to warrant rectification ($a + ur \geqslant uwz$ as in Theorem 3), the buyer will optimally "see no evil, hear no evil" and halt auditing altogether.

To assist in identifying an LVUS, we list the following result, based on Proposition 2.

Corollary 2. *Any unaudited majority-exclusive supplier will be an LVUS.*

In summary, by following the optimal policy in Theorem 4, the buyer will target whichever side of the supply network has a greater number of exclusive tier 2 suppliers and will continue trimming that side. We observe that as firms are dropped through failing audits, the network evolves toward a more balanced shape, where t_{A} and t_{B} become more similar. The retailer Target currently weighs factors such as country risk, registration status, date of last audit, and previously identified issues to select suppliers to audit (Target 2020). Our results show that the location of a supplier in the network is an important factor to consider too.

Example 2 (Continued). In more lopsided supply networks (moving rightward along each row of Table 2), there are more majority-exclusive suppliers, each with lower value to the buyer than in a more balanced network. In the spirit of Theorem 4 (applicable when the buyer audits within tier 2 only), we would expect that the buyer would audit and drop more suppliers. However, in this example with suppliers in both tiers in play, the number of suppliers audited for dropping (AD) sometimes increases as we move rightward along the rows, but it often decreases, too. Similarly, in supply networks with more shared suppliers (moving downward each column), there are fewer majority-exclusive suppliers, so we would expect the buyer would audit and drop fewer suppliers. Once again, we see that there is no monotonicity in the number of suppliers audited and dropped. We will revisit the example again to understand the opposing forces behind the fluctuations. □

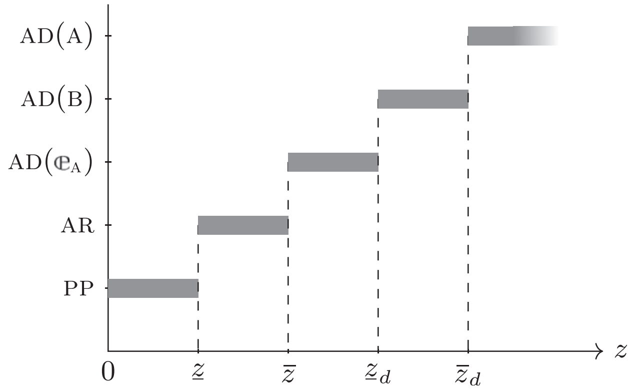
5.3. Supplier Choice When Auditing One Firm

Although we have fully characterized the optimal policy when all tier 1 firms are vetted, the problem gets substantially more complicated if the tier 1 firms are among the choices to audit. We illustrate the complexity of this problem in Section 5.4. In the current section we prove the optimal auditing policy for any state $\gamma \in \Gamma$ if we limit the buyer to audit at most one supplier (e.g., because of a limited auditing budget). Although we make this simplification to shed light on the behavior in Example 1, limiting the extent of auditing may be realistic given that we only model a single auditing cycle in an ongoing campaign, as discussed in the introduction. The more detailed version of the following result appears as Propositions D.4 and D.5 in Online Appendix D.3.

Proposition 3. *Consider a nonterminal state where the buyer can audit at most one supplier before proceeding to production. As penalty z increases, the optimal action shifts from* PP *to* AR(*i*) (*where i is any unaudited supplier*) *to* AD(*i*) (*where i is some unaudited supplier*). *Furthermore, within the interval of z where* AD(*i*) *is optimal, as z increases, the supplier i to audit shifts from firm* A*'s exclusive supplier, then to firm* B, *then to firm* A (*given that all suppliers are unaudited and firm* A *has an exclusive supplier*).

We illustrate the results of Proposition 3 in Figure 4, in which the thresholds for z are labeled $\underline{z}$, $\bar{z}$, $\underline{z}_d$, and $\bar{z}_d$. (Propositions D.4 and D.5 identify the thresholds.) With low penalty z ($z \leqslant \underline{z}$), the buyer has no incentive to make any effort to audit and proceeds directly to the production phase. With intermediate penalty z ($\underline{z} < z \leqslant \bar{z}$), retaining maximal profit from production activity remains the dominating consideration. The buyer audits a supplier but refrains from dropping it if the audit reveals noncompliance; instead, the buyer rectifies the supplier to keep it in the supply network. With high penalty z ($z > \bar{z}$), the buyer's priority shifts to auditing suppliers for dropping. In that region, if z is relatively low ($\bar{z} < z < \underline{z}_d$), the buyer will just choose an LVUS—a majority-exclusive supplier—to audit. However, once z exceeds $\underline{z}_d$, the supplier to audit is no longer an LVUS. When penalty z is intermediate ($\underline{z}_d < z \leqslant \bar{z}_d$), the buyer chooses the minority tier 1 firm B (AD(B)). The decision jeopardizes the production profit drawn from the entire side of the network dependent on firm B but may simultaneously avoid the penalty from all those suppliers. When penalty z is high ($z > \bar{z}_d$), the buyer chooses the majority tier 1 firm A with even higher stakes: the potential of losing all

Figure 4. Optimal Decision When the Buyer Audits At Most One Supplier



Notes. Schematic illustration of the optimal decision as penalty z varies and when the buyer is limited to conduct at most one audit. AR represents auditing and rectifying (if noncomplaint) any unaudited supplier.

production profit drawn from firm A's side of the supply network while avoiding penalty from this large group of suppliers.

Example 2 (Continued)**.** As we have seen earlier, contrary to our expectation set up by Theorem 4, the number of suppliers audited for dropping is not necessarily higher in more lopsided supply chains. Proposition 3 gives us a means to understand the opposing force that curbs the number of suppliers audited for dropping. A more lopsided supply chain generates less production profit for the buyer, so the violation penalty looms larger in such a supply chain. Therefore the buyer would be more willing to drop a tier 1 supplier, taking away any dependent tier 2 suppliers with it, without auditing them. As the buyer takes this aggressive approach, the total number of audits needed actually falls. All in all, as the supply network becomes more lopsided (moving rightward along the rows in Table 2), there are two opposing forces: the presence of more expendable suppliers, each of which can be audited and dropped, versus the desire to audit and drop a tier 1 supplier that takes away these expendable suppliers in one fell swoop. These two opposing forces induce undulations in the expected number of suppliers audited for dropping. □

5.4. Patterns in Optimal Auditing Paths

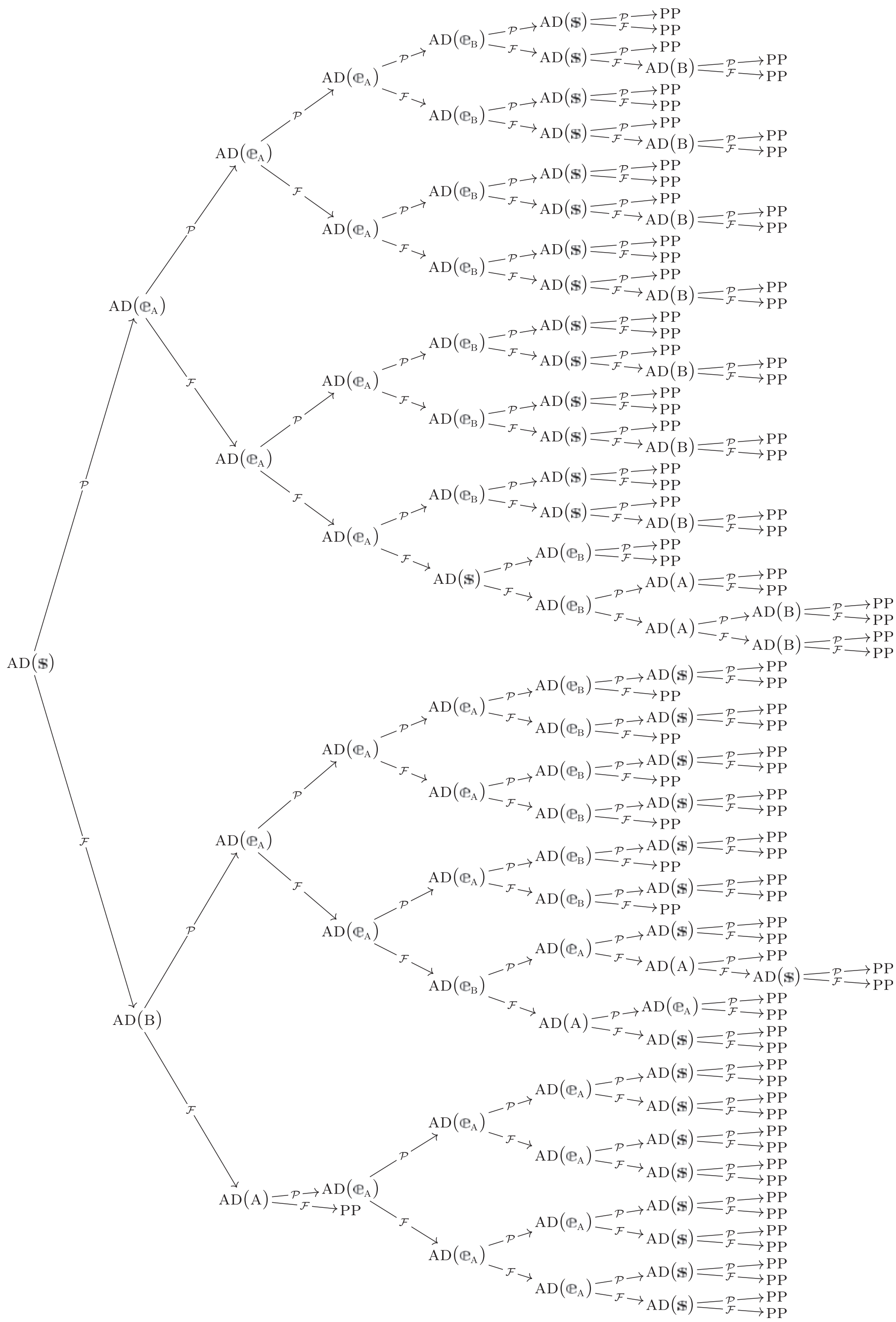
We have shown in Section 5.2 a complete picture of optimal auditing behavior in the second tier. In particular, the buyer will always prioritize an LVUS to AD. On the other hand, Example 1 shows that if we include the tier 1 firms as auditing candidates, over a certain range of parameters, the optimal first decision can be auditing and dropping (if noncompliant) a minority-exclusive supplier or even a shared supplier, neither of which is an LVUS. What drives such behavior? How can auditing and dropping (if noncompliant), say, a shared supplier, benefit the buyer more than auditing a majority-exclusive supplier?

Figure 5 presents an event tree of the buyer's optimal auditing paths in Example 1. The initial state γ may be seen in Figure 3(a). The event tree in Figure 5 is attained when penalty $z = 1500$ so that the buyer strictly prefers to first audit and drop a shared supplier (i.e., take action AD(s) at γ). In network γ, e_A may correspond to any exclusive supplier to firm A, suppliers 1, 2, or 3; e_B refers to the exclusive supplier to firm B, supplier 6; and s refers to either shared supplier, 4 or 5. We use $\mathcal{P}$ to label the branch for passing an audit and $\mathcal{F}$ for failing. Figure 5 shows an example in which the RP subphase consists only of PP activity but not AR, as a result of the high rectification cost, as discussed in Section 5.1.

The first pattern we discuss is what we call the "litmus test". In certain states, the buyer opts to audit a supplier in a pivotal position in the network (instead of an LVUS) to gauge the viability of a portion of the supply network or even the entire network. For example, the buyer chooses the shared supplier as the first firm to audit in state γ, as seen in Figure 5. Such a supplier is in a pivotal location in that it serves both firms A and B; knowing the outcome of the audit allows the buyer to deduce how valuable firms A and B will be. By testing the shared supplier early on, the buyer gains intelligence that it can act on in the early stages of auditing. In the event that the shared supplier passes, the buyer is optimistic about the viability of both sides of the supply network, and as seen in Figure 5, it takes a more cautious approach by auditing a sequence of majority-exclusive suppliers, each of which is an LVUS. Notably, however, when the shared supplier fails the audit, the buyer is pessimistic about the value of firm B, because that side of the supply network is not as attractive as it once might have been when the shared supplier was in the network. Hence, the buyer goes on to audit firm B, which is yet another litmus test: firm B is also in a pivotal position (and not an LVUS), and the buyer takes dramatically different actions depending on the outcome of that audit, as we discuss next.

We observe a second pattern, a "rescue operation" versus a "kill mission", that follows the auditing of firm B in the lower half of the event tree (AD(B)), which follows when the initial action AD(s) fails. What we label as the "kill mission" follows when firm B fails. In that case, the buyer is left with only firm A's side of

Figure 5. An Event Tree Under the Optimal Auditing Policy



the supply network with four unaudited tier 2 suppliers (there is no longer a distinction between exclusive and shared suppliers). Given the unattractive production profit of such a limited network, the buyer proceeds to audit firm A, the failing of which will kill the entire supply network, thus avoiding costly audits of all remaining tier 2 suppliers. Even if firm A passes, the buyer keeps auditing all remaining suppliers to eliminate any noncompliance.

By contrast, if firm B passes, the buyer conducts what we label a "rescue operation": after the initial setback of dropping the first shared supplier, the prospect has improved sufficiently with the passing of firm B. The buyer then mostly follows a conventional auditing sequence of LVUS suppliers in tier 2 in an attempt to only prune the less valuable suppliers.

In summary, as both the production profit and the penalty can be potentially high, the buyer takes great care in auditing. In several states the buyer's auditing choice is intended to test the waters and determine which part of the network is worthy of protection and which is not. Because a shared supplier occupies the key position of simultaneously influencing both sides of the supply network, it is the perfect candidate for the first audit as a litmus test (for the given parameters). The buyer prefers AD(𝕤) over AD(e_A) for the information AD(𝕤) can provide to guide the buyer's subsequent decision. In particular, if the first audit AD(𝕤) ascertains compliance, the buyer takes a more optimistic approach by proceeding to audit the other tier 2 suppliers, the dropping of which would not damage the buyer's production profit too much. On the other hand, if the first audit AD(𝕤) reveals noncompliance, the buyer starts to approach auditing more aggressively by directly turning to the tier 1 firms, starting with firm B. Depending on the outcome of AD(B), the buyer will conduct either a rescue operation (upon B passing the audit) or a kill mission (upon B failing the audit). The rescue operation intends to preserve the profitability of the network, whereas the kill mission has a good chance of putting an end to the network and preventing any production.

6. Extensions

6.1. Competition in the Production Phase

The production phase establishes the values of suppliers to the buyer, which facilitates the buyer's decisions in the auditing phase. Although our base model uses a specific version of Cournot competition in the production phase, our model can accommodate alternative models of competition that generate the values of suppliers. For example, in Online Appendix E we replace our Cournot competition model with the competition model in Adida et al. (2016) (ABM hereafter) for the case with a single buyer (called a "retailer" in ABM), two tier 1 firms ("intermediaries"), and any number of tier 2 firms ("suppliers"). A key difference of the model in ABM is that the sequence of decisions is reversed: the buyer chooses its quantity first, the tier 1 firms choose their quantities second, and the tier 2 firms choose their quantities last. Online Appendix E shows that replacing our competition model with ABM's competition model preserves our main results and insights.

The results in Section 5.1—the optimality of the two-subphase policy—do not depend on the mode of competition at all. Furthermore, our results on auditing in Section 5.2 continue to hold under any modes of competition possessing two intuitive properties: (1) the decreasing differences of the buyer's production profit resulting from the competition (Assumption D.1 in Online Appendix D.2) and (2) the preservation of the LVUS (Assumption D.2). The decreasing differences property says that a supplier's value to the buyer gets smaller when the network gets larger. The property reflects the substitutability of suppliers. With more competing suppliers available, the buyer relies less on each supplier. The preservation of the LVUS property says an LVUS remains an LVUS when a supplier in a different part of the network is removed. For example, consider the case where an LVUS is a majority-exclusive supplier. Removing a supplier from another part of the network only accentuates the crowding among the majority-exclusive suppliers, reinforcing the supplier's status as an LVUS. Under any competition model that exhibits the two properties, the buyer's optimal policy is to audit and drop an LVUS until it reaches the RP subphase (see the proofs in Online Appendix D.2). One can show that the ABM model possesses the two properties as well.

6.2. Heterogeneous Penalty Across Tiers

We now consider an extension where the penalty in tiers 1 and 2 can be different. Let z_1 be the penalty to the buyer arising from the exposure of a violation at a tier 1 firm, and let z_2 be the penalty from a tier 2 firm. There are several reasons why we consider the penalty to be common to firms in the same tier but possibly different across tiers. Note that firms within the same tier produce a substitutable product, likely using identical technology, which subjects them to similar violations; by contrast, suppliers in different tiers likely face different types of violations. For instance, a tier 1 parts supplier for an electronics manufacturer could be at risk for labor violations, whereas a tier 2 metal smelter could be at risk for using conflict minerals. Also, given the tier 1 supplier's proximity to the buyer, the buyer may have to contend with greater media fallout and consumer backlash as a result of a violation at a tier 1 supplier; this perspective would suggest $z_1 > z_2$.

We present the specification and results of the extension in Online Appendix F. The extension largely preserves the results from our base model, with a few interesting differences. For instance, Theorem F.1 in the online appendix shows that under the RP subphase, rather than auditing either all remaining suppliers or none, the buyer will separately treat suppliers in tier 1 and tier 2: the buyer may audit all suppliers in tier 1 but none in tier 2, or all in tier 2 but none in tier 1, because now the cost accounting differs between the two tiers. In particular, if the buyer incurs a higher penalty for violations in tier 1 ($z_1 > z_2$), the buyer may audit and rectify all tier 1 firms but carry unaudited tier 2 firms to production. As previously mentioned, $z_1 > z_2$ is a likely scenario, which provides some justification for a buyer auditing tier 1 suppliers earlier. Given the symmetry between the probability of exposure w and penalty z in the total expected penalty for the buyer ($\zeta(\gamma)$), a tier-dependent probability of exposure w_k for tier k will have the same effect on our results.

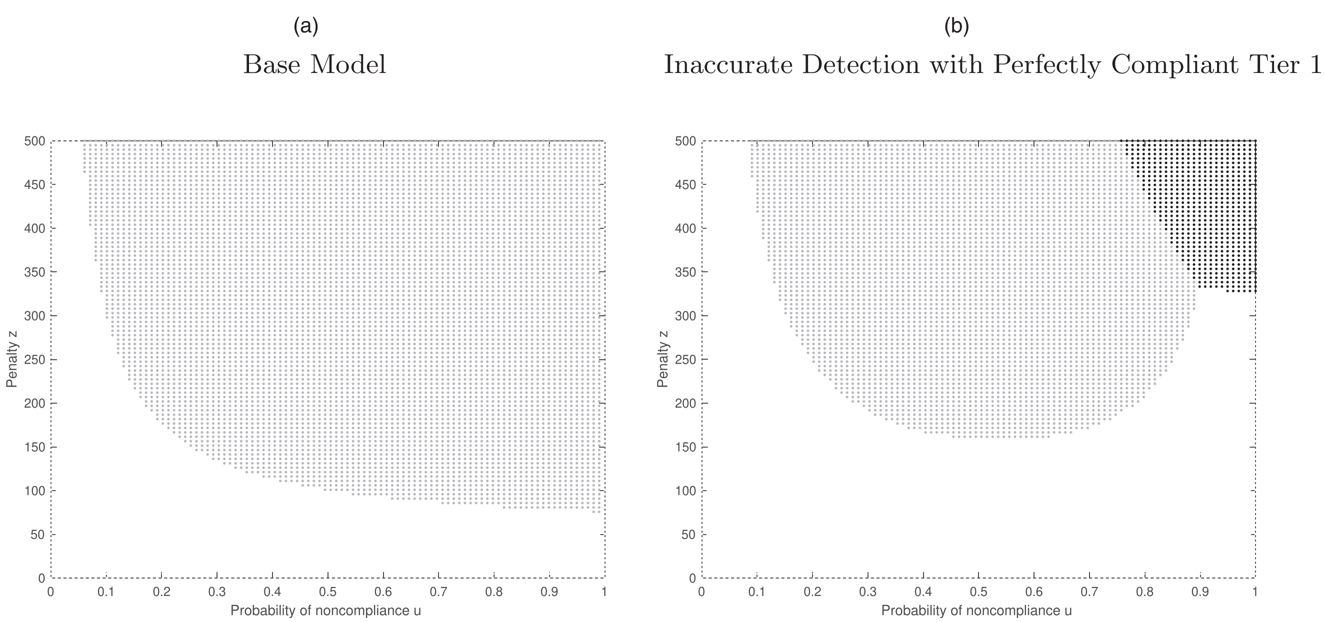
6.3. Inaccurate Detection in Auditing

We now consider an extension in which the audit is sometimes inaccurate in that it may not detect an existing noncompliance at a supplier. We present details of the extension in Online Appendix G. Here, given a supplier is noncompliant, the audit by the buyer detects the noncompliance with probability d and misses it with probability $1-d$. As a result, a supplier may still be noncompliant even after passing an audit. Let $\widehat{u}$ denote the probability that a supplier is noncompliant despite passing an audit. In parallel, we assume rectification is not always successful. Specifically, rectification lowers a supplier's noncompliance probability to $\widehat{u}$, the same as a supplier that passes an audit.

In Online Appendix G we show that our model of inaccurate detection preserves all analytical results with some minor modifications. Here, we illustrate some key qualitative departure from the base model. Consider Figure 6, which illustrates the first auditing decision in the network in Figure 1 (in the introduction) for the base model (panel (a)) and for the extension with inaccurate detection (panel (b)). The horizontal axis shows the probability of noncompliance u, and the vertical axis shows the penalty z. Within the figure, each region represents a different optimal first decision. On the left panel, the optimal decision is either to proceed to production without any auditing at all (white) or to audit and rectify any supplier (gray). On the right panel, we see the possibility of another optimal action: auditing and dropping a majority-exclusive supplier (firm 1 or firm 2) (black).

First, when audits become inaccurate, the buyer is more likely to skip auditing (the white region becomes larger). The cost of auditing is the same as before, yet the benefit is lower because the audit may miss noncompliance. As a result, the buyer is more willing to "see no evil, hear no evil". Second, consider penalty $z = 250$. In panel (a), at low values of noncompliance probability u, the firm prefers to proceed to production, tolerating any possible, yet improbable, penalties. As u increases (entering the gray region), now that violations are more likely, the firm switches to auditing and rectifying all suppliers. By contrast, in panel (b), as u increases even further, the buyer stops auditing altogether (reentering the white region). Why? In this region the buyer already feels confident that its suppliers are noncompliant (high u), yet it lacks the confidence that the audits will detect the noncompliance, so it conducts no audits (saving the costs) and proceeds to production. Third, when both u and z are high (black region in the upper right corner of panel (b)), the buyer that faces inaccurate audits would audit and drop supplier 1 (panel (b)), whereas the buyer with the benefit of accurate audits would audit and rectify any supplier (panel (a)). Under the model of inaccurate detection, a supplier that has undergone rectification may still be noncompliant,

Figure 6. Comparing Inaccurate Detection with the Base Model: Optimal First Decision in Tier 2



Notes. The optimal decision in the first step of the auditing phase in the state $\gamma = (g, U)$, where $g = (\{\text{A}, \text{B}\}, \{1, 2\}, \{4\}, \{3\})$ as in Figure 1 and $U = \{1, 2, 3, 4\}$ (i.e., the tier 2 suppliers are unaudited). The parameters are $\alpha = 400$, $\beta = 10$, $v_T = 10$, $a = 10$, $r = 20$, and $w = 0.4$. Panel (a) shows the base model where an audit accurately detects noncompliance. Panel (b) shows the model with inaccurate detection (the probability of detection $d = 0.7$) yet perfectly compliant tier 1 firms (as in the base model). We focus on optimal policies with the two-subphase structure (Theorem G.1 in the online appendix). The figure shows three regions: the white region represents PP; the gray region represents AR any unaudited supplier; and the black region represents AD(1) or AD(2).

potentially costing the buyer. Therefore, with penalty high and noncompliance likely, the buyer chooses to drop the noncompliant supplier rather than risk carrying a noncompliant supplier into production.

7. Conclusion

We study a buyer's problem of auditing suppliers for social responsibility in an existing three-tier supply network. The topology of the network determines which suppliers compete with one another, yielding a unique production equilibrium. The buyer, anticipating the equilibrium, chooses suppliers to audit in sequence and decides whether to drop or rectify a noncompliant supplier, shaping the supply network in the process.

We prove that an optimal auditing policy can be separated into two subphases: (1) the buyer audits and drops some suppliers, and (2) the buyer audits and rectifies either all remaining suppliers or none of them. One key observation is that the buyer will not know how many or which suppliers to audit until after auditing results are observed progressively. This provides one particular lesson for managers that might counter the practice of picking a predetermined set of suppliers to audit in the next cycle—for example, a year. The buyer may be better off by being agile and picking suppliers as the audits proceed. This might also have budgetary implications: the buyer may want to be flexible with the total amount to spend in auditing and rectification. Furthermore, we show that after dropping some suppliers, the buyer may find that the cost of auditing and rectifying exceeds the potential penalty, in which case the buyer may skip auditing and proceed directly to production: see no evil, hear no evil.

As the buyer audits with the intention to drop noncompliant suppliers, if it focuses exclusively on the second tier, it will always audit next an LVUS, or *least valuable unaudited supplier*, typically an exclusive supplier in the more crowded side of the network. Auditing this way thins the crowded side of the network and may, over time, render the other side of the network more crowded instead; thus the buyer might switch from one side to the other as it drops suppliers. Focusing on whichever side is more crowded, the audits tend to yield a more balanced supply network for the buyer. In practice, buyers may intend auditing suppliers based on their compliance history; our results show that the location within the network is an important factor as well.

Interestingly, when any firm in the network (including any tier 1 supplier) is a candidate for auditing, the next supplier to audit might not be an LVUS. The buyer may shift focus to a shared tier 2 supplier or even a tier 1 firm, which occupies a pivotal location within the network. We think of such audits as "litmus tests" because the outcome of such audits—whether the supplier gets dropped or not—may endow the buyer with more or less confidence over the viability of the supply network. If the pivotal supplier passes the litmus test, the buyer has more confidence over the viability of the network and may take the usual route of auditing an LVUS hereafter. If it fails, the buyer may take more drastic actions such as additional litmus tests on even more crucial suppliers. The implication for managers is clear: auditing suppliers in key locations early on can provide an important signal for the viability of the network, but the buyer must be willing to drop the supplier upon failing the audit. This would be a more appropriate course of action when the danger of violation is particularly acute, such as when the penalty and probability of exposure are high. This is part and parcel of a broader message: Buyers should be flexible in planning for their audits. They should wait to commit to the next move only after seeing the current result. Such agility may drastically change the course of action compared with a rigid auditing plan a buyer might commit to at the outset.

With two tier 1 firms in our model, there are three classes of tier 2 suppliers (exclusive to firm A, exclusive to firm B, and shared). However, as we increase the number of tier 1 suppliers, the number of supply classes of tier 2 suppliers grows quickly. For three tier 1 firms, there are seven classes, and in general, for n tier 1 firms, there will be $\sum_{i=1}^{n} C_n^i$ classes—too many to characterize the production phase equilibrium as in Propositions C.4 and C.5. The large number of supply classes also makes identifying the LVUS more challenging. However, in Online Appendix H, we are able to show the robustness of some of our auditing results for general supply networks without any restriction, if we forgo a production model altogether and instead represent production activity by an abstract production profit function for the buyer.

We next discuss a few areas of future research: (1) We study a buyer auditing an existing network, allowing the possibility of removing suppliers. Future research could consider how a buyer adds new suppliers or new links between existing suppliers. Such an addition brings the problem into the realm of network design, which requires considering geography, production and transportation costs, and quality. (2) As noted earlier, whereas we focus on the "inner" dynamics of a single auditing cycle, future work could consider a simpler representation of each cycle to allow for studying the "outer" dynamics of a multicycle horizon that incorporates history and future periods. (3) Our work does not consider different types of violations. Future work could consider how the nature of violation (even at a single supplier) affects the buyer's auditing and remediation decisions.

(4) In our model, compliance is independent across suppliers. One could consider the correlation between noncompliance among suppliers. In that case, the buyer would update its belief about noncompliance of other suppliers as auditing proceeds. (5) Starting with a network in which the upper tiers are not visible to the buyer, audits may not just uncover violations of social responsibility but also identify suppliers in upper tiers.

Acknowledgments

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Endnotes

[1] We separate the two phases, auditing followed by production, as a modeling abstraction to reflect that auditing influences the network on which production takes place.

[2] The status of a supplier reflects the buyer's assessment of the supplier's risk of violation. Even in the initial state, the buyer may consider a supplier vetted if there is sufficient confidence in the supplier's compliance as a result of, for example, the buyer having recently audited or rectified the supplier in an earlier cycle (that our model does not capture).

[3] Given $\gamma = (g, U) \in \Gamma$ and $i \in U$, $\gamma \oplus i = (g, U')$, where $U' = U \backslash \{i\}$.

[4] Given $\gamma = (g, U) \in \Gamma$, where $g = (S(1), S_{\text{A}}, S_{\text{B}}, S_{\text{AB}})$ and $i \in U$, write $\gamma \ominus i = (g', U')$. Then $U' = U \backslash D_g(i)$. If $i \in S(1)$, let $-i \in \{\text{A}, \text{B}\} \backslash \{i\}$, $S'_i = S_i$, and $S'_{-i} = S_{-i} \cup S_{\text{AB}}$, and then $g' = (S(1) \backslash D_g(i), S_{\text{A}}' \backslash D_g(i), S_{\text{B}}' \backslash D_g(i), \emptyset)$; otherwise, $g' = (S(1) \backslash D_g(i), S_{\text{A}} \backslash D_g(i), S_{\text{B}} \backslash D_g(i), S_{\text{AB}} \backslash D_g(i))$.

[5] Even though our focus is on a buyer auditing an existing supply network, we note that a powerful buyer may be able to dictate tier 1 firms' choices of tier 2 suppliers when *designing a new supply network*.

[6] At the state $\gamma_{\emptyset}$, which corresponds to the null supply network $g_{\emptyset}$, we set the buyer's production profit to zero: when the buyer has access to no supplier, there is no production activity and thus no production profit.

[7] We rephrase Proposition 1 as Proposition C.6 in Online Appendix C.3, where we derive $\theta(t_{\text{B}}, t_{\text{AB}})$.

[8] On a different note, in Online Appendix B, we discuss how the expected number of suppliers audited and the probability of exposure change with u, the probability of noncompliance.

[9] For the purposes of this section, it is useful to separately identify a boundary element of the state space, denoted as $\gamma_1 = (g, U)$, the state in which the buyer is served by two separate linear branches (i.e., $g = (\{\text{A}, \text{B}\}, \{1\}, \{2\}, \emptyset)$), and both tier 1 firms are vetted and both tier 2 suppliers are unaudited (i.e., $U = \{1, 2\}$). Network γ_1 does not satisfy Condition 1 (hence, we treat it separately) because it is a special case: the removal of any supplier will prune an entire branch of the network, eliminating the competition between firms A and B. Fortunately, we know the optimal auditing policy at γ_1, which we fully describe in Theorem D.2 in Online Appendix D.2.

[10] Note that "LVUS" is pronounced |ˈɛlvəs|, the same as, the "King of Rock and Roll."

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