Appendices

A. Asymmetric Downstream Markets

When the downstream firms are asymmetric in terms of idiosyncratic valuations, cross- and self-price or capability sensitivities, and adoption cost functions, a stylized treatment of the game discussed in this paper is not practical. In this section, we discuss the numerical analysis of 2,3,4-firm downstream tiers facing linear demand functions - (L) - asymmetric in a subset of these factors. The details of the algorithm (LDCEA) used to solve the equilibrium of the whole game is provided in Appendix A.1.

We show that the majority of the properties of the upstream return function and downstream technological potential previously seen in the symmetrical system are preserved.

We construct the exact upstream leader’s return curve, \( R(M^1) \), for all \( M^1 \geq M^2 \), in addition to the optimal premium slope, \( a^*(M^1) \), equilibrium downstream capability levels \( Q^*_j \), and downstream profits \( \pi^*_j \) as a function of \( M^1 \). The results of these asymmetric cases are then compared to the symmetric base cases from which they are derived. Due to the immense number of possibilities of asymmetries, we do not give a complete account for how different asymmetry types affect different equilibrium variables. We point out some observations which are robust even under multiple types of asymmetry. A representative example is provided below.

Example:

\( n = 4, M^2 = 25, \alpha = (30 40 40 50)^T, \kappa = (3 2 2.5 2)^T, Q^0_j = 5 \forall j, c = (0.1 0.1 0.1 0.15). \)

\[
\beta = \begin{pmatrix}
  -0.6 & 0.3 & 0.3 & 0.3 \\
  0.3 & -0.8 & 0.3 & 0.3 \\
  0.3 & 0.2 & -0.6 & 0.2 \\
  0.2 & 0.2 & 0.4 & -0.8
\end{pmatrix},
\gamma = \begin{pmatrix}
  2.2 & -0.6 & -0.6 & -0.6 \\
  -0.5 & 1.9 & -0.4 & -0.4 \\
  -0.5 & -0.4 & 2 & -0.4 \\
  -0.5 & -0.4 & -0.6 & 2.1
\end{pmatrix}.
\]

The TP of this 4-firm downstream tier is \( \tau \sim (42.94 \ 43.38 \ 59.13 \ 65.46)^T \).
**Observation 1.** The upstream leader’s return is a quasi-concave increasing function in $M^1$ between the upstream laggard capability ($M^2$) and maximum pass-through level ($Q(M^2)$), and constant for capability levels above $Q(M^2)$.

**Observation 2.** If the downstream firms are asymmetric in some parameter, the optimal premium slope $a^*$ is not necessarily a monotonic function of upstream capability (Figure 1(b)). The upstream leader may increase or reduce $a^*$ with its increasing capability. This results in non-monotonic levels of adoption by downstream firms (Figure 1(d)). Downstream firms’ equilibrium profits are also possibly not monotonic in the upstream leader’s capability (Figure 1(e)).

**Observation 3.** If the downstream firms are symmetric in all senses, the optimal premium slope is a monotonic decreasing function of upstream leader’s capability and the downstream firms’ adoption levels and profits are monotonically increasing in the upstream leader’s capability.
Table 1: Optimal premium slope, equilibrium capabilities and profits.

<table>
<thead>
<tr>
<th>$M^1$</th>
<th>$a^*(M^1)$</th>
<th>$Q^+(M^1)$</th>
<th>$\pi^+$</th>
<th>$R(M^1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>0.000</td>
<td>(25,000 25,000 25,000 25,000)</td>
<td>(4823.74 5397.54 5059.28 5457.52)</td>
<td>0.00</td>
</tr>
<tr>
<td>27.5</td>
<td>1.304</td>
<td>(27.500 25,000 27.500 27.500)</td>
<td>(4969.98 5649.03 5224.13 5623.51)</td>
<td>0.19</td>
</tr>
<tr>
<td>30</td>
<td>1.096</td>
<td>(30,000 25,000 30,000 30,000)</td>
<td>(5088.23 5856.61 5371.72 5779.27)</td>
<td>1.17</td>
</tr>
<tr>
<td>32.5</td>
<td>0.995</td>
<td>(32.500 25,000 32.500 32.499)</td>
<td>(5178.29 6022.56 5502.01 5924.80)</td>
<td>6.61</td>
</tr>
<tr>
<td>35</td>
<td>1.094</td>
<td>(35,000 25,000 35,000 30.834)</td>
<td>(5291.76 6266.41 5635.09 6081.09)</td>
<td>10.94</td>
</tr>
<tr>
<td>37.5</td>
<td>0.881</td>
<td>(37.500 26,020 37.500 35.735)</td>
<td>(5178.29 6308.94 5709.92 6167.50)</td>
<td>14.87</td>
</tr>
<tr>
<td>40</td>
<td>0.780</td>
<td>(39.840 28.091 40.000 38.369)</td>
<td>(5225.65 6374.14 5734.04 6230.87)</td>
<td>20.94</td>
</tr>
<tr>
<td>42.5</td>
<td>0.819</td>
<td>(39.537 27.602 42.500 37.727)</td>
<td>(5327.66 6501.34 5762.63 6339.77)</td>
<td>24.27</td>
</tr>
<tr>
<td>45</td>
<td>0.742</td>
<td>(40.658 29.188 45.000 39.807)</td>
<td>(5262.75 6532.79 5741.15 6387.84)</td>
<td>27.50</td>
</tr>
<tr>
<td>47.5</td>
<td>0.671</td>
<td>(41.567 30.593 46.454 41.705)</td>
<td>(5158.77 6511.57 5703.55 6387.84)</td>
<td>29.21</td>
</tr>
<tr>
<td>50</td>
<td>0.671</td>
<td>(41.567 30.593 46.454 41.705)</td>
<td>(5158.77 6511.57 5703.55 6387.84)</td>
<td>32.44</td>
</tr>
<tr>
<td>52.5</td>
<td>0.671</td>
<td>(41.567 30.593 46.454 41.705)</td>
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<tr>
<td>55</td>
<td>0.671</td>
<td>(41.567 30.593 46.454 41.705)</td>
<td>(5158.77 6511.57 5703.55 6387.84)</td>
<td>34.34</td>
</tr>
<tr>
<td>57.5</td>
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<td>(41.567 30.593 46.454 41.705)</td>
<td>(5158.77 6511.57 5703.55 6387.84)</td>
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<tr>
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<td>0.671</td>
<td>(41.567 30.593 46.454 41.705)</td>
<td>(5158.77 6511.57 5703.55 6387.84)</td>
<td>34.34</td>
</tr>
</tbody>
</table>

For most practical purposes, the return to a superior upstream technology from the downstream tier implied by the capability dimension, $R(M^1)$, can be approximated with a concave or quasi-concave increasing function up to a capability level (which is lower than the highest element of the technological potential) and constant for any capability level beyond. Any technology possessed above this level, $\bar{Q}(M^2)$, is “held-up” in the upper tier. This approximation is very similar to the function depicted in Figure 2(a) for the symmetrical case. Thus, the symmetrical analysis constructed in §4.3 is a good proxy for the asymmetric case.

A.1. LDCEA Algorithm (For §A)

1. Initialization: Define an indicator state vector $s$ for the downstream capability equilibrium $\ni s_j =$

$$
\begin{align*}
1 & \text{ if } Q_j < Q_j^1, \\
2 & \text{ if } Q_j = Q_j^1, \\
3 & \text{ if } Q_j \in (Q_j^0, M^2], \\
4 & \text{ if } Q_j = M^2, \\
5 & \text{ if } Q_j \in (M^2, M^1], \\
6 & \text{ if } Q_j = M^1.
\end{align*}
$$

Initialize the set $S$ of all possible states ($6^n$ elements). Also define:

\[
J = \begin{pmatrix} 1 & \ldots & 1 \\ 1 & \ldots & 1 \\ 1 & \ldots & 1 \end{pmatrix}_{n \times n}, \quad I(s \geq i) = \begin{pmatrix} 1_{(s_1 \geq i)} \\ \vdots \\ 1_{(s_n \geq i)} \end{pmatrix}, \quad \kappa = \begin{pmatrix} \kappa_1 & 0 & \ldots \\ 0 & \kappa_j & 0 & \ldots \\ 0 & \ldots & 0 & \kappa_n \end{pmatrix},
\]

\[
c = (c_1 \ldots c_j \ldots c_n)^T.
\]
\[
\text{diag}(\mathbb{1}(s \geq 5)) = \begin{pmatrix}
\mathbb{1}(s_1 \geq 5) & \ldots & 0 \\
0 & \mathbb{1}(s_j \geq 5) & 0 \\
0 & \ldots & \mathbb{1}(s_n \geq 5)
\end{pmatrix}.
\]

To be used for later in check for the possibility of multiple equilibria under any state \(s\), also create the vectors \(dm(s)\), \(dD(s)\); and the matrices \(\nabla m(s)\), and \(\nabla D(s)\):

\[
\begin{align*}
\text{dm}(s)_{j} &= \tilde{\gamma}_{jj} + a(\tilde{d}_{jj} - 1)\mathbb{1}(s_j \geq 5), \\
\text{dD}(s)_{j} &= \hat{\gamma}_{jj} + a\hat{d}_{jj}\mathbb{1}(s_j \geq 5), \\
\nabla m(s)_{jj} &= 0, \\
\nabla D(s)_{jj} &= 0,
\end{align*}
\]

\[
\begin{align*}
\nabla m(s)_{jk} &= \tilde{\gamma}_{jk} + a\tilde{d}_{jk}\mathbb{1}(s_k \geq 5), \\
\nabla D(s)_{jk} &= \hat{\gamma}_{jk} + a\hat{d}_{jk}\mathbb{1}(s_k \geq 5).
\end{align*}
\]

2. For all \(s \in S\):

i. Check if multiple equilibria is expected. If

\[
\text{dm}(s) \ast (\nabla D(s).e) + \text{dD}(s) \ast (\nabla m(s).e) - \text{dm}(s) \ast \text{dD}(s) + 2\kappa > 0
\]

holds, there is a unique equilibrium. Otherwise, there may be multiple.

ii. Create the candidate equilibrium with \(Q^{\text{cand}}(s) = A^{-1}(s) \cdot b(s)\), where:

\[
A_j(s) = \begin{cases} 
\Upsilon_j(s) & \text{if } s_j \in \{1, 3, 5\}, \\
e_j & \text{if } s_j \in \{2, 4, 6\}.
\end{cases}
\]

\[
b_j(s) = \begin{cases} 
Z_j(s) & \text{if } s_j \in \{1, 3, 5\}, \\
Q^0_j & \text{if } s_j = 2, \\
M[2] & \text{if } s_j = 4, \\
M[1] & \text{if } s_j = 6,
\end{cases}
\]
where

\[ \Upsilon(s) = dp(s) \ast \hat{\gamma} + dD(s) \ast \tilde{\gamma} - 2 \mathbb{I}(s \geq 3) \ast \kappa \]

\[ + adp(s) \ast \tilde{d} \ast (\text{diag}(\mathbb{1}(s \geq 5) \cdot J))^\top \]

\[ + adD(s) \ast \tilde{d} \ast (\text{diag}(\mathbb{1}(s \geq 5) \cdot J))^\top \]

\[ - a \text{diag}(dD(s) \ast \mathbb{1}(s \geq 5)), \]

\[ (3) \]

\[ Z(s) = dD(s) \ast c + c_u \cdot dD(s) - dp(s) \ast \hat{\alpha} - dp(s) \ast \hat{\epsilon} - dD(s) \ast \tilde{\alpha} - dD(s) \ast \tilde{\epsilon} \]

\[ + aM^2 dp(s) \ast \tilde{d} \mathbb{1}(s \geq 5) \]

\[ + aM^2 dD(s) \ast \tilde{d} \mathbb{1}(s \geq 5) \]

\[ - aM^2 dD(s) \ast \mathbb{1}(s \geq 5) \]

\[ (4) \]

where

\[ dp(s) = \text{Diag}(\hat{\gamma}) + a \left( \text{Diag}(\tilde{d}) - e \ast \mathbb{1}(s \geq 5) \right), \]

\[ dD(s) = \text{Diag}(\hat{\gamma}) + a \text{Diag}(\tilde{d}) \ast \mathbb{1}(s \geq 5) \]

iii. Check consistency of FOCs for \( Q^{\text{cand}}(s) \).

For all \( j \):

i. if \( s_j = 2 \) check if \( \zeta_j^1(s_{-j}, Q^{\text{cand}}) \geq 0 \) and \( \zeta_j^3(s_{-j}, Q^{\text{cand}}) \leq 0 \),

ii. if \( s_j = 4 \) check if \( \zeta_j^3(s_{-j}, Q^{\text{cand}}) \geq 0 \) and \( \zeta_j^5(s_{-j}, Q^{\text{cand}}) \leq 0 \),

iii. if \( s_j = 6 \) check if \( \zeta_j^5(s_{-j}, Q^{\text{cand}}) \geq 0 \),

iv. if \( s_j = 1 \), check if \( Q^{\text{cand}}_j < Q^0_j \),

v. if \( s_j = 3 \), check if \( Q^{\text{cand}}_j \in (Q^0_j, M^{[2]}) \),

vi. if \( s_j = 5 \), check if \( Q^{\text{cand}}_j \in (M^{[2]}, M^{[1]}) \).

where

\[ \zeta_j^k(s_{-j}, Q) = \Upsilon(s_j = k, s_{-j}) \cdot Q - Z(s_j = k, s_{-j}), \quad \forall k \in \{1, 3, 5\}. \]

\[ (5) \]

If sustained for all the elements of \( \vec{s} \), then record \( Q^{\text{cand}}(s) \). Otherwise discard.

\[ B. \text{ Proofs of Some Propositions and Corollaries} \]

**Proof of Proposition** \[ \]
(G) We need to show that $\mathbf{p}$ that simultaneously solves the FOCs in (3) exists, is unique, and returns non-negative profit to each downstream firm. Let $p^*_j(p_{-j})$ denote the marginal best responses which satisfy the FOC in (3) of the concave problem:

$$\max_{p_j} (p_j - c_j - c_u - \delta(Q_j))D_j(Q, p). \quad (6)$$

Since $\partial D_j(Q, p)/\partial p_j \leq 0$, $p^*_j(p_{-j}) \geq c_j + c_u + \delta(Q_j)$. Hence, the price best response which also returns non-negative profit, denote it with $p_j(p_{-j})$, is equal to the marginal price response, i.e., $p_j(p_{-j}) = p^*_j(p_{-j})$.

By implicit derivation the slope of marginal price response of a firm $j$ to that of firm $k$, $k \neq j$:

$$\frac{\partial p_j(p_{-j})}{\partial p_j} = -\frac{\partial D_j(Q, \mathbf{p})}{\partial p_k} + (p_j - c_j - c_u - \delta(Q_j))\frac{\partial^2 D_j(Q, \mathbf{p})}{\partial p_j \partial p_k} \quad (7)$$

Under Assumption 1(a) and (b): $\partial p_j(p_{-j})/\partial p_k \geq 0, \forall j, k \neq j$. Hence, $p^*(Q|\delta(\cdot))$ exists and is unique.

(L) For the demand form in Equation (1) FOCs in (3) can be reorganized into:

$$\alpha_j + \beta_j \cdot \mathbf{p} + \beta_{jj} \hat{\mathbf{p}} + \gamma_j \cdot \mathbf{Q} - \beta_{jj}(c_j + c_u) - \beta_{jj}\delta(Q_j) = 0, \forall j. \quad (8)$$

Define $B = \beta + \text{Diag}(\beta) \cdot \mathbf{I}$, $C = \text{Diag}(\beta) \ast (\mathbf{e} + c_u \cdot \mathbf{I})$, and $\nabla(Q) = (\beta_{11}\delta(Q_1) \ldots \beta_{jj}\delta(Q_j) \ldots \beta_{nn}\delta(Q_n))^T$. Then, (8) can be rewritten in the following matrix form:

$$\alpha + B\mathbf{p} + \gamma \mathbf{Q} - C - \nabla(Q) = 0. \quad (9)$$

Solving for $\mathbf{p}$ ($B$ is invertible under Assumption 1):

$$p^*(Q|\delta(\cdot)) = -B^{-1}\alpha - B^{-1}\gamma \mathbf{Q} + B^{-1}C + B^{-1}\nabla(Q), \quad (10)$$

where $B^{-1}\nabla(Q) = B^{-1} \ast [\text{Diag}(\beta) \ast \mathbf{e}]^{-1} = \hat{\mathbf{d}}\delta(Q)$.

(M) For the demand form in Equation (2) FOCs in (3) can be rewritten as:

$$D_j(Q, \mathbf{p}) + (p_j - c_j - c_u - \delta(Q_j))\frac{\partial D_j(Q, \mathbf{p})}{\partial Q_j} = 0,$$

$$D_j(Q, \mathbf{p})(1 - \beta_{jj}(p_j - c_j - c_u - \delta(Q_j))) = 0,$$
Solving the last for \( p_j \) provides the expression in (5).

**Proof of Corollary 1** Follows by replacing \( p \) in \( D(Q, p) \) with the expression in (4) for (L) and with the one in (3) for (M).

**Proof of Proposition 2** Given \( p_j^*(Q | \delta(\cdot)) \) and \( D_j^*(Q | \delta(\cdot)) \) defined in Proposition 1 and Corollary 1 for any feasible \( Q \) optimal (which does not guarantee positive profit) best response of downstream firm \( j \) should solve the following problem:

\[
\max_{Q_j} m_j^*(Q | \delta(\cdot))D_j^*(Q | \delta(\cdot)) - (K_j(Q_j) - K_j(Q_j^0))^+.
\] (11)

Let the best response that solves the problem be \( Q_j(Q_{-j}) \). The problem objective in (11) is not continuously differentiable at \( Q_j^0 \) because of the investment term - and at \( M^2 \) because of the premium function and. Two marginal revenues defined (6) and (7) are valid for \( Q_j < M^2 \) and \( Q_j \geq M^2 \). Under Assumption 2 (c) and convex increasing investment costs, the objective function is unimodal. Then, we can write the 6 different FOCs for \( Q_j(Q_{-j}) \) depending on the 6 possible regions in each of which the objective is continuous and differentiable. \( Q^*(\delta(\cdot)) \) becomes the simultaneous solution of a set of FOCs, one for each firm.

Next we show under Assumption 2 (a) and (b) the best responses are non-increasing and result in a unique equilibrium. The slope - first partial derivatives - of the capability best responses:

\[
\frac{\partial Q_j(Q_{-j})}{\partial Q_k} = \begin{cases} 
- \frac{\partial p_j^0(Q | \delta(\cdot))/\partial Q_k}{\partial p_j^0(Q | \delta(\cdot))/\partial Q_j} & \text{if } Q_j(Q_{-j}) < Q_j^0, \\
- \frac{\partial p_j^0(Q | \delta(\cdot))/\partial Q_j}{\partial p_j^0(Q | \delta(\cdot))/\partial Q_k} & \text{if } Q_j(Q_{-j}) \in (Q_j^0, M^2), \\
- \frac{\partial p_j^0(Q | \delta(\cdot))/\partial Q_j}{\partial p_j^0(Q | \delta(\cdot))/\partial Q_k} & \text{if } Q_j(Q_{-j}) \in (M^2, M^1).
\end{cases}
\] (12)

At points \( Q_j^0, M^2, \) and \( M^1 \), the slope of the best responses are either 0 or one of the neighboring values. Denominators in (12) are all negative under Assumption 2 (c). Numerators are also negative under Assumption 2 (a). Hence, \( \frac{\partial Q_j(Q_{-j})}{\partial Q_k} \) are non-positive.

The spectral radius of the matrix of the first partials of the capability best responses - denote it with \( J(Q) \) - such that \( J(Q)_{jj} = 0 \) and \( J(Q)_{jk} = \frac{\partial Q_j(Q_{-j})}{\partial Q_k} \) for \( k \neq j \), is less than 1 if the row sums of the same matrix are strictly higher -1. The sum of the \( j^{th} \) row of \( J(Q) \) matrix is greater than

\[
- \frac{\sum_{k \neq j} \partial p_j^0(Q | \delta(\cdot))/\partial Q_k}{\partial p_j^0(Q | \delta(\cdot))/\partial Q_j} & \text{if } Q_j(Q_{-j}) \leq M^2, \\
- \frac{\sum_{k \neq j} \partial p_j^0(Q | \delta(\cdot))/\partial Q_k}{\partial p_j^0(Q | \delta(\cdot))/\partial Q_j} & \text{if } Q_j(Q_{-j}) > M^2.
\] (13)
which are both strictly greater than $-1$ under Assumption $2$ (a)-(c) . This establishes the uniqueness of $\hat{Q}(\delta(\cdot))$.

Finally, Assumption $2$ (c) implies that if $Q_j(Q^*_j) \geq 0$, it is positive, continuous and decreasing in any $Q_k$, $k \neq j$. Hence, there is no discontinuity in best responses, which guarantees the existence of a non-negative profit equilibrium for all downstream firms.

**Proof of Corollary 3** If $Q_j(0) \leq M^2$, then one of the first 3 cases in Proposition $2$ is uniquely satisfied for each downstream firm for $\delta(Q) = 0, \forall Q$. $\rho_j^0(Q| \delta(\cdot))$ is decreasing in $\partial \delta(Q)/\partial Q, \forall j$.

**Proof of Corollary 4** Under symmetry:

\begin{align*}
\rho(Q, a) &= \partial ((p^*(Q| a) - c_j - a(Q - M^2)) D^*(Q| a)) / \partial Q_j \\
\rho(Q, a) &= \frac{1}{\nu_{jj}} \partial D_j(Q| a) / \partial Q_j.
\end{align*}

**Proof of Proposition 3** From the Definition in $[1]$, TP is defined by the 3rd case for every downstream firm in Proposition $2$. Hence, $\tau = \{ Q | \rho(Q, 0) = 0 \}$. Solving this equation for $\rho(Q, a)$ as defined in Corollary $4$ for $a = 0$ gives the expressions in $[12]$ and in $[13]$, respectively.

**Proof of Proposition 4** Signs of the partials follow from the closed form expressions below.

For the symmetric linear demand case (LS):

\begin{align*}
\frac{\partial \tau}{\partial \alpha_1} &= -\beta_1(2\beta_1 \gamma_{11} + \beta_{12}((n - 2)\gamma_{11} - (n - 1)\gamma_{12})) / \\
(\kappa_1(2\beta_{11} - \beta_{12})(2\beta_{11} + (n - 1)\beta_{12})^2 + \beta_{11}(\gamma_{11} + (n - 1)\gamma_{12})(2\beta_{11}\gamma_{11} + \beta_{12}((n - 2)\gamma_{11} - (n - 1)\gamma_{12}))
\end{align*}

\begin{align*}
\frac{\partial \tau}{\partial \gamma_{11}} &= \beta_{11} (\beta_1 (2\beta_{11} \gamma_{11} + \beta_{12} (\gamma_{11}(n - 2) - \gamma_{12}(n - 1))) \\
&\quad (\beta_{12} (\gamma_{12} (n^2 - 4n + 3) + 2\gamma_{11}(n - 2)) + 2\beta_{11} (2\gamma_{11} + \gamma_{12}(n - 1))) \\
&\quad - (2\beta_{11} + \beta_{12}(n - 2)) ((2\beta_{11} - \beta_{12}) \kappa_1 (2\beta_{11} + \beta_{12}(n - 1))^2 + \beta_{11} (\gamma_{11} + \gamma_{12}(n - 1)) \\
&\quad (2\beta_{11}\gamma_{11} + \beta_{12} (\gamma_{11}(n - 2) - \gamma_{12}(n - 1))) ) \\
&\quad (\alpha_1 + (c_0 + c_1)(\beta_{11} + \beta_{12}(n - 1))) / \\
&\quad ((2\beta_{11} - \beta_{12}) \kappa_1 (2\beta_{11} + \beta_{12}(n - 1))^2 + \beta_{11} (\gamma_{11} + \gamma_{12}(n - 1)) (2\beta_{11}\gamma_{11} + \beta_{12} (\gamma_{11}(m - 2) - \gamma_{12}(n - 1))))^2
\end{align*}
\[
\frac{\partial \tau}{\partial \beta_{12}} = \\
\beta_{11} ((2\beta_{11} \gamma_{11} + \beta_{12} (\gamma_{11}(n-2) - \gamma_{12}(n-1))) \\
(-\kappa_{1} (2\beta_{11} + \beta_{12}(n-1))^2 + 2(2\beta_{11} - \beta_{12}) \kappa_{1}(n-1) (2\beta_{11} + \beta_{12}(n-1)) \\
+ \beta_{11} (\gamma_{11} + \gamma_{12}(n-1)) (\gamma_{11}(n-2) - \gamma_{12}(n-1))) \\
(\alpha_{1} + (c_{u} + c_{1}) (\beta_{11} + \beta_{12}(n-1))) - (\gamma_{11}(n-2) - \gamma_{12}(n-1)) \\
((2\beta_{11} - \beta_{12}) \kappa_{1} (2\beta_{11} + \beta_{12}(n-1))^2 + \beta_{11} (\gamma_{11} + \gamma_{12}(n-1)) (2\beta_{11}\gamma_{11} + \beta_{12} (\gamma_{11}(n-2) - \gamma_{12}(n-1))))
\]

\[
\frac{\partial \tau}{\partial \gamma_{12}} = \beta_{11} (\beta_{11}(n-1) (2\beta_{11} \gamma_{11} + \beta_{12} (\gamma_{11}(n-3) - 2\gamma_{12}(n-1))) \\
(2\beta_{11} \gamma_{11} + \beta_{12} (\gamma_{11}(n-2) - \gamma_{12}(n-1))) - \beta_{12}(1-n) ((2\beta_{11} \\
- \beta_{12}) \kappa_{1} (2\beta_{11} + \beta_{12}(n-1))^2 + \beta_{11} (\gamma_{11} + \gamma_{12}(n-1)) (2\beta_{11}\gamma_{11} + \beta_{12} (\gamma_{11}(n-2) - \gamma_{12}(n-1)))) \\
(\alpha_{1} + (c_{u} + c_{1}) (\beta_{11} + \beta_{12}(n-1))) \\
/ ((2\beta_{11} - \beta_{12}) k_{1} (2\beta_{11} + \beta_{12}(n-1))^2 + \beta_{11} (\gamma_{11} + \gamma_{12}(n-1)) (2\beta_{11}\gamma_{11} + \beta_{12} (\gamma_{11}(n-2) - \gamma_{12}(n-1))))^2
\]

\[
\frac{\partial \tau}{\partial \beta_{11}} = \\
\frac{2\beta_{11} \gamma_{11} (\alpha_{1} + (c_{u} + c_{1}) (\beta_{11} + \beta_{12}(n-1))) \\
(2\beta_{11} - \beta_{12}) k_{1} (2\beta_{11} + \beta_{12}(n-1))^2 + \beta_{11} (\gamma_{11} + \gamma_{12}(n-1)) (2\beta_{11}\gamma_{11} + \beta_{12} (\gamma_{11}(n-2) - \gamma_{12}(n-1))) \\
(2\beta_{11}\gamma_{11} + \beta_{12} (\gamma_{11}(n-2) - \gamma_{12}(n-1))) - \beta_{12}(1-n) ((2\beta_{11} \\
- \beta_{12}) \kappa_{1} (2\beta_{11} + \beta_{12}(n-1))^2 + \beta_{11} (\gamma_{11} + \gamma_{12}(n-1)) (2\beta_{11}\gamma_{11} + \beta_{12} (\gamma_{11}(n-2) - \gamma_{12}(n-1)))) \\
(\alpha_{1} + (c_{u} + c_{1}) (\beta_{11} + \beta_{12}(n-1))) \\
+ (\beta_{11} (2\beta_{11}\gamma_{11} + \beta_{12} (\gamma_{11}(n-2) - \gamma_{12}(n-1))) (2\beta_{11}\gamma_{11} + \beta_{12} (\gamma_{11}(n-2) - \gamma_{12}(n-1)))) (2\beta_{11} + \beta_{12}(n-1))^2 \\
+ 4(2\beta_{11} - \beta_{12}) k_{1} (2\beta_{11} + \beta_{12}(n-1)) (2\beta_{11}\gamma_{11} (\gamma_{11} + \gamma_{12}(n-1))) + (\gamma_{11} + \gamma_{12}(n-1)) \\
(2\beta_{11}\gamma_{11} + \beta_{12} (\gamma_{11}(n-2) - \gamma_{12}(n-1)))) \\
(\alpha_{1} + (c_{u} + c_{1}) (\beta_{11} + \beta_{12}(n-1)))) \\
/ ((2\beta_{11} - \beta_{12}) k_{1} (2\beta_{11} + \beta_{12}(n-1))^2 + \beta_{11} (\gamma_{11} + \gamma_{12}(n-1)) (2\beta_{11}\gamma_{11} + \beta_{12} (\gamma_{11}(n-2) - \gamma_{12}(n-1))))^2 \\
- \beta_{11} (c_{u} + c_{1}) (2\beta_{11}\gamma_{11} + \beta_{12} (\gamma_{11}(n-2) - \gamma_{12}(n-1))) \\
(2\beta_{11} - \beta_{12}) k_{1} (2\beta_{11} + \beta_{12}(n-1))^2 + \beta_{11} (\gamma_{11} + \gamma_{12}(n-1)) (2\beta_{11}\gamma_{11} + \beta_{12} (\gamma_{11}(n-2) - \gamma_{12}(n-1)))) \\
(2\beta_{11}\gamma_{11} + \beta_{12} (\gamma_{11}(n-2) - \gamma_{12}(n-1)))
\]
For the symmetric multiplicative demand case (MS):

\[
\frac{\partial \tau}{\partial \kappa_1} = \beta_11 (2\beta_11 - \beta_12) (2\beta_11 + \beta_12(n - 1))^2 (2\beta_11 \gamma_11 + \beta_12 (\gamma_11(n - 2) - \gamma_12(n - 1)))
\]

\[
(\alpha_1 + (c_u + c_1) (\beta_11 + \beta_12(n - 1))) /
\]

\[
((2\beta_11 - \beta_12) \kappa_1 (2\beta_11 + \beta_12(n - 1))^2 + \beta_11 (\gamma_11 + \gamma_12(n - 1)) (2\beta_11 \gamma_11 + \beta_12 (\gamma_11(n - 2) - \gamma_12(n - 1))))^2
\]

For the symmetric multiplicative demand case (MS):

\[
\frac{\partial \tau}{\partial \lambda_1} = \frac{1}{\lambda_1 (\vartheta_11 + k_1 + \vartheta_12(n - 1))}
\]

\[
\frac{\partial \tau}{\partial \nu_11} = \left( - \log \left( \frac{\lambda_1 \vartheta_11}{\nu_11} \right) - (n - 1) \log \left( 1 - e^{\beta_11 \vartheta_11} \right) \right)
\]

\[
+ \nu_11 (c_u + c_1) + \log (g_1) + \frac{\gamma_11 + k_1 + \vartheta_12(n - 1)}{\vartheta_11} + \log (k_1) + 1 \right) \right) / (\vartheta_11 + k_1 + \vartheta_12(n - 1))^2
\]

\[
\frac{\partial \tau}{\partial \nu_{12}} = \frac{(n - 1) (\nu_11 c_1 + \nu_11 c_u + 1)}{\nu_11 \left( e^{\beta_12 \left( \frac{1}{\vartheta_11} + c_u + c_1 \right) - 1} \right)} \right) \right) / (\vartheta_11 + k_1 + \vartheta_12(n - 1))^2
\]

\[
\frac{\partial \tau}{\partial \vartheta_12} = - \frac{(n - 1) \left( \log \left( \frac{\lambda_1 \vartheta_11}{\nu_11} \right) + (n - 1) \log \left( 1 - e^{\beta_11 \vartheta_11} \right) \right) - \nu_11 (c_u + c_1) - \log (g_1) - \log (k_1) - 1}{(\vartheta_11 + k_1 + \vartheta_12(n - 1))^2}
\]

\[
\frac{\partial \tau}{\partial \vartheta_{11}} = \left( - \log \left( \frac{\lambda_1 \vartheta_11}{\nu_11} \right) - (n - 1) \log \left( 1 - e^{\beta_11 \vartheta_11} \right) \right)
\]

\[
+ \nu_11 (c_u + c_1) + \log (g_1) + \frac{\vartheta_11 + k_1 + \vartheta_12(n - 1)}{\vartheta_11} + \log (k_1) + 1 \right) \right) / (\vartheta_11 + k_1 + \vartheta_12(n - 1))^2
\]

\[
\frac{\partial \tau}{\partial k_1} = \left( - \log \left( \frac{\lambda_1 \vartheta_11}{\nu_11} \right) - (n - 1) \log \left( 1 - e^{\beta_11 \vartheta_11} \right) \right)
\]

\[
+ \nu_11 (c_u + c_1) + \log (g_1) - \frac{\vartheta_11 + k_1 + \vartheta_12(n - 1)}{k_1} + \log (k_1) + 1 \right) \right) / (\vartheta_11 + k_1 + \vartheta_12(n - 1))^2
\]

C. Auxiliary Material for Symmetric Downstream Tier

C.1. Approximating Upstream Return

The maximum net contribution an upstream leader can receive from a capability level of \( M_1 - M_1 \in [M_2, \tau] \) - from a symmetric tier of \( n \) firms can be described with the following mathematical problem where
$Q'(a)$ is the symmetric unconstrained downstream capability equilibrium, i.e.,

$$Q'(a) = \left\{ Q \left| \rho^0(Q|a) = \frac{\partial K(Q)}{\partial Q} \lor \rho^1(Q|a) = \frac{\partial K(Q)}{\partial Q} \right. \right\}.$$ 

Only one of the two conditions hold depending on the value of $a$. For small values of $a$, second one is likely to hold.

$$R(M^1) = \max_a n \cdot a(M^1 - M^2)D^*(M^1|a) \quad (14)$$

$$s.t: M^1 \leq M^1,$$ \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} (15)

$$Q'(a) \geq M^1.$$ \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} (16)

where $M^1$ is the capability level offered and adopted. (Constraint (16) ensures that this is the case.)

We define two pseudo revenue functions: $R''(M^1)$ is the upstream revenue when it induces the adoption of all of its capability. (Constraint (15) is made to hold strictly) $R'(M^1)$ is the upstream revenue when it induces the adoption of all of its capability and charges the highest premium possible. (Constraints (15) and (16) are made to hold strictly.) The two approximations can be written as the solution of the following

![Figure 2: Approximating $R(M^1)$ at $\bar{Q}(M^2)$ using two functions: $R''(M^1)$ and $R'(M^1)$.](image-url)
problems.

\[ R''(M^1) = \max_a n a(M^1 - M^2)D^*(M^1 | a) \]  
\( s.t: M^1 = M^1 \),  
\[ Q'(a) \geq M^1. \]  
\[ R'(M^1) = \max_a n a(M^1 - M^2)D^*(M^1 | a) \]  
\( s.t: M^1 = M^1 \),  
\[ Q'(a) \geq M^1. \]  
\[ R'(M^1) = \max_a n a(M^1 - M^2)D^*(M^1 | a) \]  
\( s.t: M^1 = M^1 \),  
\[ Q'(a) \geq M^1. \]

Given the identical objective functions and tighter constraints: \( R(M^1) \geq R''(M^1) \geq R'(M^1) \), for all \( M^1 \in [M^1, \tau] \).

**Lemma 1.** \( R(M^1) \) defined as the solution of (14) - (16) is quasi-linear and non-decreasing in \( M^1 \).

**Proof** Let the feasible region of the problem in (14) - (16) for any \( M^1 \) be denoted by \( \Xi(M^1) \). \( M^1 \leq M^1 \).

Hence, for any \( M^a \) and \( M^b \) such that \( M^a < M^b \), \( \Xi(M^b) \supseteq \Xi(M^a) \). Hence, the optimal value of the objective is non-decreasing in \( M^1 \): \( R(M^b) \geq R(M^a) \).

**Lemma 2.** For any \( M^2 \leq \tau \), there exists a capability level - denote it with \( \bar{Q}(M^2) \) - such that \( R(M^1) = R(\bar{Q}(M^2)) \) for all \( M^1 \geq \bar{Q}(M^2) \). Moreover \( \bar{Q}(M^2) < \tau \).

**Proof** Define \( M^1^*(M^1) \) as the optimal value for the problem (14) - (16). We need to show that the constraint (15) is not binding for \( M^1 \geq \bar{Q}(M^2) \) for a threshold \( \bar{Q}(M^2) \) strictly less than \( \tau \).

Suppose otherwise. Then for some \( M^2 \), \( M^1^*(\tau) = \tau \). But, \( Q'(a) \geq \tau \Rightarrow a^* = 0 \land R(M^1) = 0 \). There exists \( \epsilon \geq 0 \) such that \( a'(\tau - \epsilon) > 0 \) and \( D^*(\tau - \epsilon | a'(\tau - \epsilon)) > 0 \). Contradiction for optimality.

Lemma 2 describes the maximum technology that can pass from the upper tier to the lower tier as a function of the upstream laggard capability level. It also shows that it is strictly less than \( \tau \). It has the following corollary result.

**Corollary 1.**

1. For all \( M^2 \leq \tau \), \( \max_{M^1 \in [M^2, \tau]} R(M^1) = R(\bar{Q}(M^2)) \).
2. \( \arg \max_{M^1} R(M^1) \supseteq [\bar{Q}(M^2), \tau] \).
3. \( R''(\bar{Q}(M^2)) = R(\bar{Q}(M^2)) \).
**Proof** Result (1) and (2) follow directly from Lemma 1 and Lemma 2. Result (3) follow from the fact that Problems (.14)-(.16) and (.17)-(1.19) become identical for \( M^1 = \bar{Q}(M^2) \) since at optimality Constraint (15) is binding.

**Lemma 3.** \( R''(M^1) = R'(M^1) \) for all \( M^1 \geq \bar{Q}(M^2) \).

**Proof** Results from the fact that for \( M^1 \geq \bar{Q}(M^2) \), Constraint (19) is binding. Suppose it were not. Note that for both problems that define \( R''(M^1) \) and \( R'(M^1) \), \( M^1 = M^1 \). Then in the optimal solution of (17)-19 \( a^*(M^1) < a'(M^1) \). Then, \( \exists M^x > M^1 \) such that \( a^*(M^1) = a'(M^x) \) as \( a'(M^1) \) is strictly decreasing in \( M^1 \). But, then the objective in (17) can attain a higher value at \( na^*(M^1)(M^x - M^2)D^*(M^x | a^*(M^1)) \). Contradiction.

**Corollary 2.** \( R'(\bar{Q}(M^2)) = R(\bar{Q}(M^2)) \).

**Proof** Follows from Lemma 3 and Corollary 1.

### C.2. Fixed Technological Potential Numerical Analysis (For §5.2)

<table>
<thead>
<tr>
<th>Factor pairs (LS)</th>
<th>( Q^* )</th>
<th>( D^* )</th>
<th>( p^* )</th>
<th>( \delta^* )</th>
<th>( R^* )</th>
<th>( \pi^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>((\alpha_1, \gamma_{12}))</td>
<td>-0.0043</td>
<td>0.0407</td>
<td>0.0951</td>
<td>0.3567</td>
<td>0.6016</td>
<td>0.4981</td>
</tr>
<tr>
<td>((\alpha_1, \beta_{11}))</td>
<td>0.0010</td>
<td>0.0421</td>
<td>0.0291</td>
<td>0.1683</td>
<td>0.3364</td>
<td>0.2812</td>
</tr>
<tr>
<td>((\alpha_1, \kappa_{1}))</td>
<td>0.0069</td>
<td>0.0378</td>
<td>0.0902</td>
<td>0.8613</td>
<td>0.8828</td>
<td>0.4317</td>
</tr>
<tr>
<td>((\gamma_{11}, \gamma_{12}))</td>
<td>0.1313</td>
<td>-0.6234</td>
<td>-0.2034</td>
<td>1.1390</td>
<td>0.2165</td>
<td>-2.0234</td>
</tr>
<tr>
<td>((\gamma_{11}, \beta_{11}))</td>
<td>-0.0087</td>
<td>0.1046</td>
<td>-0.5850</td>
<td>-0.7155</td>
<td>-0.6758</td>
<td>-0.8599</td>
</tr>
<tr>
<td>((\gamma_{11}, \kappa_{1}))</td>
<td>-0.0274</td>
<td>0.6625</td>
<td>0.4851</td>
<td>0.0600</td>
<td>0.7321</td>
<td>1.2316</td>
</tr>
<tr>
<td>((\beta_{12}, \gamma_{12}))</td>
<td>0.0104</td>
<td>0.0008</td>
<td>0.0566</td>
<td>0.2594</td>
<td>0.2606</td>
<td>0.0843</td>
</tr>
<tr>
<td>((\beta_{12}, \beta_{11}))</td>
<td>-0.0323</td>
<td>0.8737</td>
<td>-0.1314</td>
<td>-0.3736</td>
<td>0.2836</td>
<td>3.9794</td>
</tr>
<tr>
<td>((\beta_{12}, \kappa_{1}))</td>
<td>-0.0368</td>
<td>0.9533</td>
<td>0.7789</td>
<td>0.3730</td>
<td>1.4422</td>
<td>4.9132</td>
</tr>
</tbody>
</table>

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Table .3: Equilibrium of the (MS) base case under the modification of different tuples for fixed TP

<table>
<thead>
<tr>
<th>Modification on Base Case</th>
<th>$Q^*$</th>
<th>$D^*$</th>
<th>$p^*$</th>
<th>$\delta^*$</th>
<th>$R^*$</th>
<th>$\pi^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(\lambda_1, \nu_{12}) = (491.3774, 0.01)$</td>
<td>9.1308</td>
<td>2.1567</td>
<td>1.7467</td>
<td>0.1967</td>
<td>2.1211</td>
<td>2.6901</td>
</tr>
<tr>
<td>$(\lambda_1, \nu_{12}) = (4885.0, 0.10)$</td>
<td>6.7408</td>
<td>17.671.034</td>
<td>1.7027</td>
<td>0.2527</td>
<td>1348.9769</td>
<td>2208.7627</td>
</tr>
<tr>
<td>$(\lambda_1, \nu_{12}) = (68.406, 0.01)$</td>
<td>7.3664</td>
<td>3.7475</td>
<td>11.7066</td>
<td>1.4066</td>
<td>26.3568</td>
<td>37.4706</td>
</tr>
<tr>
<td>$(\lambda_1, \nu_{12}) = (25230, 359, 1.3)$</td>
<td>7.5955</td>
<td>61.7278</td>
<td>1.1811</td>
<td>0.1119</td>
<td>34.5303</td>
<td>47.4783</td>
</tr>
<tr>
<td>$(\lambda_1, \nu_{12}) = (158.5617, 0.01)$</td>
<td>7.5775</td>
<td>2.9494</td>
<td>1.7313</td>
<td>0.1813</td>
<td>2.4428</td>
<td>3.3672</td>
</tr>
<tr>
<td>$(\lambda_1, \nu_{12}) = (35825, 3808, 0.10)$</td>
<td>7.5787</td>
<td>267.8892</td>
<td>1.7314</td>
<td>0.1814</td>
<td>243.9237</td>
<td>334.8501</td>
</tr>
<tr>
<td>$(\lambda_1, g_{1}) = (500.0, 0.001)$</td>
<td>7.5783</td>
<td>3.7390</td>
<td>1.7314</td>
<td>0.1814</td>
<td>3.3912</td>
<td>4.6733</td>
</tr>
<tr>
<td>$(\lambda_1, g_{1}) = (50000.0, 0.10)$</td>
<td>7.5783</td>
<td>373.8984</td>
<td>1.7314</td>
<td>0.1814</td>
<td>339.1172</td>
<td>467.3269</td>
</tr>
<tr>
<td>$\delta_{111} = (0.3857, 0.01)$</td>
<td>7.3524</td>
<td>143.4139</td>
<td>1.6207</td>
<td>0.0707</td>
<td>50.6702</td>
<td>179.2630</td>
</tr>
<tr>
<td>$\delta_{112} = (0.0877, 0.0700)$</td>
<td>7.8722</td>
<td>12.7287</td>
<td>1.8664</td>
<td>0.3164</td>
<td>20.1397</td>
<td>15.9061</td>
</tr>
<tr>
<td>$\delta_{113} = (0.3947, 0.10)$</td>
<td>8.6573</td>
<td>413.0920</td>
<td>10.8095</td>
<td>0.5095</td>
<td>1052.3510</td>
<td>4130.9158</td>
</tr>
<tr>
<td>$\delta_{114} = (0.1636, 1.00)$</td>
<td>7.8664</td>
<td>20.4767</td>
<td>1.4787</td>
<td>0.1878</td>
<td>18.3065</td>
<td>20.4719</td>
</tr>
<tr>
<td>$\delta_{115} = (0.3217, 0.01)$</td>
<td>6.9504</td>
<td>46.1582</td>
<td>1.6439</td>
<td>0.0939</td>
<td>21.6646</td>
<td>57.6970</td>
</tr>
<tr>
<td>$\delta_{116} = (0.0927, 0.10)$</td>
<td>8.5667</td>
<td>23.0530</td>
<td>1.8905</td>
<td>0.3405</td>
<td>39.2426</td>
<td>28.8027</td>
</tr>
<tr>
<td>$\delta_{117} = (0.3070, 0.001)$</td>
<td>7.0095</td>
<td>45.3510</td>
<td>1.6514</td>
<td>0.1014</td>
<td>23.0019</td>
<td>56.6883</td>
</tr>
<tr>
<td>$\delta_{118} = (0.0668, 0.100)$</td>
<td>8.9025</td>
<td>17.9576</td>
<td>1.9493</td>
<td>0.3993</td>
<td>35.8553</td>
<td>22.3910</td>
</tr>
<tr>
<td>$\delta_{119} = (0.15880, 0.01)$</td>
<td>9.1387</td>
<td>2.4764</td>
<td>1.7469</td>
<td>0.1969</td>
<td>2.4381</td>
<td>3.0897</td>
</tr>
<tr>
<td>$\delta_{1110} = (0.9812, 0.06)$</td>
<td>7.2862</td>
<td>74.4147</td>
<td>1.7235</td>
<td>0.1735</td>
<td>64.5498</td>
<td>93.0140</td>
</tr>
<tr>
<td>$\delta_{1111} = (0.0406, 1.00)$</td>
<td>7.6903</td>
<td>5.4043</td>
<td>11.7175</td>
<td>1.4715</td>
<td>39.7630</td>
<td>54.0384</td>
</tr>
<tr>
<td>$\delta_{1112} = (0.8072, 1.00)$</td>
<td>7.5565</td>
<td>45.5454</td>
<td>1.4447</td>
<td>0.1474</td>
<td>32.9517</td>
<td>45.5413</td>
</tr>
<tr>
<td>$\delta_{1113} = (0.2429, 0.0001)$</td>
<td>7.6543</td>
<td>2.9511</td>
<td>1.7331</td>
<td>0.1831</td>
<td>2.7018</td>
<td>3.6881</td>
</tr>
<tr>
<td>$\delta_{1114} = (3.0083, 0.10)$</td>
<td>7.3758</td>
<td>216.8287</td>
<td>1.7261</td>
<td>0.1761</td>
<td>190.9524</td>
<td>271.0250</td>
</tr>
<tr>
<td>$\delta_{1115} = (0.2685, 0.0010)$</td>
<td>7.6491</td>
<td>4.0653</td>
<td>1.7330</td>
<td>0.1830</td>
<td>3.7205</td>
<td>5.0812</td>
</tr>
<tr>
<td>$\delta_{1116} = (2.7011, 0.0700)$</td>
<td>7.3817</td>
<td>211.2620</td>
<td>1.7263</td>
<td>0.1763</td>
<td>187.9550</td>
<td>266.5470</td>
</tr>
</tbody>
</table>

Table .4: $\Delta$ ratios in the (MS) model

<table>
<thead>
<tr>
<th>Factor pairs (MS)</th>
<th>$Q^*$</th>
<th>$D^*$</th>
<th>$p^*$</th>
<th>$\delta^*$</th>
<th>$R^*$</th>
<th>$\pi^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(\lambda_1, \vartheta_{12})$</td>
<td>-0.27</td>
<td>8.3608</td>
<td>-0.0003</td>
<td>-0.0023</td>
<td>6.4875</td>
<td>8.3788</td>
</tr>
<tr>
<td>$(\lambda_1, \nu_{11})$</td>
<td>0.0001</td>
<td>0.0422</td>
<td>-0.0025</td>
<td>-0.0025</td>
<td>0.0008</td>
<td>0.0007</td>
</tr>
<tr>
<td>$(\lambda_1, k_{1})$</td>
<td>1</td>
<td>0.4376</td>
<td>0</td>
<td>0</td>
<td>0.4378</td>
<td>0.4376</td>
</tr>
<tr>
<td>$(\lambda_1, g_{1})$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$(\vartheta_{11}, \vartheta_{12})$</td>
<td>-0.0997</td>
<td>1.2849</td>
<td>-0.2138</td>
<td>-0.9044</td>
<td>0.8496</td>
<td>1.2849</td>
</tr>
<tr>
<td>$(\delta_{11}, \nu_{11})$</td>
<td>0.1220</td>
<td>-0.6384</td>
<td>-0.5798</td>
<td>-0.4360</td>
<td>-0.6600</td>
<td>-0.6683</td>
</tr>
<tr>
<td>$(\vartheta_{11}, k_{1})$</td>
<td>-0.4035</td>
<td>0.8686</td>
<td>-0.2603</td>
<td>-0.5583</td>
<td>-1.4079</td>
<td>0.8690</td>
</tr>
<tr>
<td>$(\vartheta_{11}, g_{1})$</td>
<td>-0.4440</td>
<td>0.9931</td>
<td>-0.2966</td>
<td>-8.2800</td>
<td>-0.9187</td>
<td>0.9947</td>
</tr>
<tr>
<td>$(\nu_{12}, \vartheta_{12})$</td>
<td>-0.0391</td>
<td>5.6082</td>
<td>-0.0026</td>
<td>-0.0230</td>
<td>4.9183</td>
<td>5.6188</td>
</tr>
<tr>
<td>$(\nu_{12}, \nu_{11})$</td>
<td>-0.0016</td>
<td>0.6809</td>
<td>-0.0804</td>
<td>-0.0827</td>
<td>-0.0157</td>
<td>-0.0144</td>
</tr>
<tr>
<td>$(\nu_{12}, k_{1})$</td>
<td>-0.0032</td>
<td>6.3664</td>
<td>-0.0004</td>
<td>-0.0033</td>
<td>6.1207</td>
<td>6.3675</td>
</tr>
<tr>
<td>$(\nu_{12}, g_{1})$</td>
<td>-0.0039</td>
<td>5.6784</td>
<td>-0.0004</td>
<td>-0.0041</td>
<td>5.4643</td>
<td>5.6782</td>
</tr>
</tbody>
</table>
Table 5: Equilibrium of the (LS) base case under the modification of different tuples for fixed TP

<table>
<thead>
<tr>
<th>Modification on Base Case</th>
<th>$Q^*$</th>
<th>$D^*$</th>
<th>$p^*$</th>
<th>$\delta^*$</th>
<th>$R^*$</th>
<th>$\pi^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>($\alpha_1^<em>, \gamma_1^</em>) = (19.549, -0.15)$</td>
<td>73.0388</td>
<td>111.9471</td>
<td>193.5856</td>
<td>8.3403</td>
<td>3734.6834</td>
<td>7367.4160</td>
</tr>
<tr>
<td>($\alpha_1^<em>, \gamma_1^</em>) = (294.59, -0.8)$</td>
<td>68.6449</td>
<td>176.0367</td>
<td>452.5459</td>
<td>50.1912</td>
<td>35341.9847</td>
<td>58996.0950</td>
</tr>
<tr>
<td>($\alpha_1^<em>, \beta_1^</em>) = (11.20, -0.65)$</td>
<td>72.3613</td>
<td>104.0770</td>
<td>185.0831</td>
<td>6.1661</td>
<td>2567.0162</td>
<td>5499.5166</td>
</tr>
<tr>
<td>($\alpha_1^<em>, \beta_1^</em>) = (210.5603, -1.0)$</td>
<td>73.6882</td>
<td>181.9722</td>
<td>280.6947</td>
<td>24.6056</td>
<td>17910.1652</td>
<td>32971.6373</td>
</tr>
<tr>
<td>($\gamma_1^<em>, \gamma_1^</em>) = (1.9196, -0.1)$</td>
<td>74.0542</td>
<td>122.5575</td>
<td>230.1785</td>
<td>13.6841</td>
<td>6708.3423</td>
<td>12778.8521</td>
</tr>
<tr>
<td>($\gamma_1^<em>, \beta_1^</em>) = (2.6858, -0.5)$</td>
<td>78.1513</td>
<td>90.3705</td>
<td>210.4021</td>
<td>20.2503</td>
<td>7320.1350</td>
<td>1887.9365</td>
</tr>
<tr>
<td>($\gamma_1^<em>, \beta_1^</em>) = (1.4914, -0.5)$</td>
<td>74.9680</td>
<td>113.9197</td>
<td>332.9445</td>
<td>26.2013</td>
<td>11939.3703</td>
<td>20859.4416</td>
</tr>
<tr>
<td>($\gamma_1^<em>, \beta_1^</em>) = (2.7616, -1.0)$</td>
<td>74.4055</td>
<td>124.2530</td>
<td>164.3299</td>
<td>9.9442</td>
<td>4942.3992</td>
<td>5305.1688</td>
</tr>
<tr>
<td>($\gamma_1^<em>, \kappa_1^</em>) = (0.1156, 0.0)$</td>
<td>73.7144</td>
<td>114.0755</td>
<td>208.1552</td>
<td>11.2225</td>
<td>5120.8690</td>
<td>8846.4616</td>
</tr>
<tr>
<td>($\gamma_1^<em>, \beta_1^</em>) = (0.2217, -0.25)$</td>
<td>75.1300</td>
<td>121.4614</td>
<td>247.2391</td>
<td>18.3557</td>
<td>7918.0330</td>
<td>12806.0963</td>
</tr>
<tr>
<td>($\gamma_1^<em>, \beta_1^</em>) = (0.0762, -0.50)$</td>
<td>76.1448</td>
<td>81.3997</td>
<td>238.6590</td>
<td>18.8899</td>
<td>6150.5295</td>
<td>3369.6529</td>
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<tr>
<td>($\beta_1^<em>, \kappa_1^</em>) = (0.1427, 0.2)$</td>
<td>75.5792</td>
<td>91.6824</td>
<td>186.9860</td>
<td>13.9278</td>
<td>5107.7649</td>
<td>4414.4513</td>
</tr>
<tr>
<td>($\beta_1^<em>, \kappa_1^</em>) = (0.2127, 2.65)$</td>
<td>74.6740</td>
<td>120.1659</td>
<td>234.4992</td>
<td>15.6209</td>
<td>7508.3955</td>
<td>11482.7103</td>
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</tbody>
</table>